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Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment

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ABSTRACT

Negotiations between a country in default and its international creditors are modeled as a dynamic game in an environment of weak contractual enforcement. The country cannot borrow internationally until it settles with all creditors. Delay arises in equilibrium as creditors engage in strategic hold-up. The model affirms the conventional wisdom that delay increases with more creditors, and with the advent of "vulture" creditors. Contrary to conventional wisdom, putting collective action clauses into bond contracts may increase delay via free-riding on negotiation costs, even while preventing strategic holdup and reducing total negotiation costs. Secondary debt markets consolidate debt with high—and disperse debt with low—creditor bargaining power. Whether secondary markets reduce or increase delay, depends on the interaction between strategic holdup and debt consolidation effects. The analysis contributes to the theory of multi-player dynamic timing games through a general treatment of the comparative dynamics used to answer key applied questions about sovereign debt negotiation.

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1 Introduction

Negotiations to restructure sovereign debt are time consuming, taking an average of more than seven years to complete. Delays of this length are puzzling because they are very costly to all parties: sovereign debtors face disruptions in their access to international capital markets when in default, while creditors suffer large losses in the value of their investments.

Why is it so difficult to restructure in a timely manner? In light of a number of recent highly publicized cases, policymakers have emphasized the role of holdout creditors in delaying the restructuring process and have proposed the introduction of a range of mechanisms designed to facilitate collective action by creditors. These have included collective action clauses (CACs) in bond contracts, which bind holdouts to a super-majority's settlement. However, there is at best ambiguous evidence—from the pricing of sovereign bonds—that mechanisms like CACs have led market participants to expect faster debt restructurings in the future[1].

In this paper, we present a model of holdout creditors that captures the main features of sovereign debt restructuring. In the model, each member of a group of creditors must individually decide when to engage and negotiate with a sovereign debtor in default. Enforcement of the sovereign's contractual commitments is weak: Among other things, there is no international court to impose direct penalties for non-compliance, or that can be used to prevent sovereigns from caving-in to holdouts. Creditors are only able obtain settlements by threatening to disrupt a sovereign's access to international finance markets. Such weak enforcement generates a timing game between creditors, who vie to settle later than each other in the hope of extracting a higher settlement from the sovereign: We refer to this as *strategic holdup*—the main cause of delay in our model. When all creditors are symmetric, we find that as their number rises, competition to be the last to settle intensifies, and delay is increased. Moreover, when we extend the analysis to an asymmetric case to allow for "vulture creditors"—who are either better bargainers or more patient—we find that their

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[1] Eichengreen and Mody (2000, 2004) find that the presence of collective action clauses raise borrowing costs for low rated issuers, and lower costs for high rated issuers. By contrast, Becker, Richards and Thaicharoen (2003) find that the effect on borrowing costs is typically both economically and statistically insignificant. Gugiatti and Richards (2003, 2004) document the startling degree of ignorance of market participants as to the contractual provisions of bonds in their portfolio's.

presence also adds to delay. This holds true even if vultures' bargaining power is the result of legal tactics that generate higher costs than those faced by normal creditors.

Perhaps our most striking result is that a number of policy proposals aimed at accelerating resolution, such as CACs, may actually increase delay. This is because the costs of bargaining are borne primarily by creditors who engage the sovereign early in the process, while under a CAC all creditors receive the same financial settlement. As a result, while CACs eliminate strategic holdup, they may lead some creditors to delay engagement with the sovereign in order to *free ride* on negotiation costs. We show that delay may increase even when the costs of negotiating under a CAC are small (indeed, even smaller than without a collective action clause) because it is the *incidence* of costs that matters rather than their size. Thus, CACs increase delay when the resulting free rider effect outweighs the strategic holdup effect. When the two forces are roughly equal in magnitude, we are able to explain the ambiguous empirical findings on the pricing of bonds with CACs.

Many have argued that the emergence of secondary markets for bonds will exacerbate delay in restructuring by increasing the dispersion of bond-holdings. To examine this, we model the emergence of a secondary market by allowing creditors to endogenously consolidate or disperse their bond-holdings until the total market value of debt is maximized. We show that when creditor bargaining power is high, equilibrium in the secondary market has fewer debt-holders, as creditors consolidate holdings to reduce delay caused by strategic holdout. The impact of the emergence of secondary markets on delay then depends on the initial fragmentation of debt holdings and the level of bargaining power. If debt is initially closely-held, and bargaining power is low, creditors will sell off part of their holdings leading to greater delay. Conversely, if debt is widely held and bargaining power is high, bond-holdings will be consolidated and delay will fall.

After the literature review of section 1.A and our justification of the key assumptions in section 2, in section 3 we present a two-creditor example with a simple bargaining protocol which is able to capture many of the key features of interest. Section 4 develops a general version of this model with an arbitrary number of creditors and a number of alternative bargaining protocols. We apply a recursive procedure to solve the general model for a symmetric Markov perfect equilibrium, and then find the effect on model dynamics of: arbitrary changes in payoff profiles (including, e.g., rises bargaining power under various protocols); increasing the number of creditors by adding a player and a payoff of any magnitude; CACs with $N > 2$ creditors and a $100\gamma\%$ supermajority rule; secondary debt markets in the general case; and, finally, an asymmetric version of the model with N vultures and M normal creditors. To the best of our knowledge, the model of weak contractual enforcement, its application to the sovereign debt restructuring problem, and the analysis of the comparative dynamics of the many player timing game, are all novel.

1.A Literature

There is an emergent literature which models the outcome of a sovereign default as a game between a sovereign debtor and its creditors. YUE (2006), BI (2007), and BENJAMIN AND WRIGHT (2008) assume that creditors are able to perfectly coordinate when negotiating with the sovereign. KLETZER (2002), HALDANE ET AL (2003), WEINSCHELBAUM AND WYNNE (2005), and WRIGHT (2001, 2005) all study creditor coordination in models that abstract from delay. Our paper differs from this literature in that it studies how *imperfect* creditor coordination generates delay in bargaining. BOLTON AND JEANNE (2005) study the decision of a sovereign to issue debt that is exogenously either 'easy' or 'hard' to restructure. In our model, delay—our metric for the ease of restructuring—is endogenous, and we seek to explain how the pattern of delay varies with the renegotiation environment. PITCHFORD AND WRIGHT (2008) uses numerical methods to study the effect of different debt renegotiation environments on the sovereign's level of borrowing, its default decision, and welfare. In contrast, the current paper is a theoretical analysis of the determinants of delay for a sovereign already in default. Finally, BRONER, MARTIN AND VENTURA (2007) describe an environment in which the presence of secondary markets eliminates a sovereign's incentive to default. In our environment, secondary markets may either increase or decrease the costs of default depending on the bargaining power of creditors.

Our paper is related to the law and economics literature on multi-plaintiff settlement (see, for example the surveys by SPIER (2007) and DAUGHETY AND REINGANUM (2005)). In the canonical model, a defendant bargains with many plaintiffs with the goal of minimizing transfers in a zero-sum setting. In our framework, a creditor who has not agreed to a settlement can interfere with access to international capital markets, and so prevent the sovereign from realizing a productive opportunity.[2] Che and Spier (2008) use a static model in which plaintiffs enjoy scale economies in litigation, to study the normative effects of different legal mechanisms (e.g. voting, covenants or transfer payments). In contrast, our paper is a positive study of the determinants of delay that is inherently dynamic. Other papers in this literature assume that a broad set of claims are enforceable to study contractual solutions to the hold-up problem (see, for example, Spier (2003a) and (2003b), and Daughety and Reinganum (2003) who study the use of 'Most Favored Nation' (MFN) clauses). Such enforceability is credible in the domestic context. In the international context of sovereign debt negotiation, the enforcement of claims against a sovereign is limited. To capture this we assume that the only credible and binding commitment a sovereign can make to its creditors is an immediate cash payment or its equivalent.

The MFN settlement literature typically assumes private information on the part of plaintiffs. There is also an extensive literature on bargaining under imperfect information (see for example, Grossman and Perry (1986), Bai and Zhang (2008) for an application to sovereign debt, and the survey by Fudenburg, Levine and Tirole (1985) among many others). With private information, delay is source of strategic information revelation. The availability of extensive information about sovereign countries leads us to assume complete and symmetric information, and to focus on lack of commitment as a source of delay.

Finally, by studying the decision of a creditor as to *when* to negotiate, our paper is related to the literatures on timing games and hold-up. Menezes and Pitchford (2004), analyze a problem in which a buyer seeks to purchase two objects from two distinct sellers and show that delay increases as the degree of complementarity between the objects rises. In our model, either unanimous or super-majority agreement by creditors is required for a settlement, so that our environment assumes a high degree of complementarity. Our comparative dynamic results, which can be applied to more general multi-player continuous-time games of hold-out, extends the characterization of equilibria in the literature on timing

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[2] The avoidance of court costs is another motive explored in the pretrial settlement literature–and one which is clearly productive albeit in a different way to the current paper. See the paper by Spier (1992) for the pioneering analysis of the single plaintiff case.

games and wars of attrition such as Hendricks, Weiss and Wilson (1988), Haig and Cannings (1989), Bulow and Klemperer (1999), Kapur (1995) and many others.

2 Background

In this section, we survey the salient features of the environment in which sovereign debt restructuring negotiations take place and describe how these features motivate the construction of our theoretical framework. We then illustrate the main results that arise in our framework using a series of simple examples. The general model, along with detailed derivations of these results, are presented in the succeeding sections.

We argue that sovereign debt negotiations take place in a "weak contractual environment" that is characterized by five key features.

(I) *Sovereigns lack the ability to commit to contracts*

There is no international equivalent of a domestic bankruptcy court. The sovereign's inability to commit to a stream of payments historically arose from the *doctrine of sovereign immunity* that gave governments immunity from legal action in both their own jurisdictions and in those of creditor countries. Although this doctrine has been weakened with the passage of the Foreign Sovereign Immunity Act of 1976—which allowed lawsuits against sovereigns—it remains very difficult for creditors to collect on favorable judgments. Numerous cases support this observation. For example, in *Allied Bank International v. Banco Credito Agricola de Cartago (1985)*[3] while Allied successfully argued against Costa Rica's defense that default was an 'Act of State', it ultimately received only the same compensation as other creditors. Even when assets are outside a nation's borders, seizure can be difficult. A spectacular example is of the Swiss trading firm *Noga's* many failed attempts to seize Russian assets as various as sailing ships, jet fighter planes, uranium shipments, embassy bank accounts and art exhibits.[4]

(II) *Creditors can delay or disrupt a sovereign's access to new capital*

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[3] *11757 F.2d 516 (2d Cir. 1985).*

[4] See, for example, "Pride of Russia's navy set to remain in the dock" *Financial Times*, 22nd July 2000, p.8 (on naval sailing ship), "Russian fighters dodge Swiss creditors" *Financial Times*, 23rd July 2002, p.8 (on jet fighters), "Clinton Issues Executive Order to Protect HEU Assets from Lawsuits" *Nuclear Fuel*, 26th June, 2000, p.16 (on uranium shipments), "French court set to rule on frozen Russian funds" *Financial Times* 7th August 2000, p.3 (on embassy bank accounts), "Paintings Returned to Russia" *New York Times*, 24th November 2005, p.2 (on art exhibits), and the survey in Wright (2001).

Historically, the London Stock Exchange refused to list a sovereign's new money bonds until it had settled with all creditors, blocking or at least disrupting the ability of sovereigns in default to issue new debt. This is reflected in an absence of borrowing by defaulted countries in the historical record (Tomz 2007). Since the passage of the Foreign Sovereign Immunity Act, a variety of newer legal tactics have been used to disrupt credit market access, the most famous of which concerns the efforts of the vulture fund Elliott Associates to block Peru's servicing of debts issued as part of its 1993 restructuring.[5] Such lawsuits have become increasingly common: The World Bank and International Monetary Fund (2007) report forty-seven court cases against a total of eleven highly indebted poor countries; Following it's 2001 default, there were over one hundred lawsuits launched against Argentina in creditor country jurisdictions (Gelpern 2005).[6]

The combination of sovereigns' inability to commit (I), and creditors' ability to disrupt or delay capital market access (II), has left limited means by which a sovereign can credibly repay its creditors. Thus,

(III) *Settlement payments are typically by immediate cash payment, or its equivalent.*

The inability of sovereigns to commit to future repayments means that restructuring agreements typically involve a large cash component. For example, in Peru's 1993 restructuring, about 4bn was a cash payment with a further 4bn in face value of new debt. The latter had a lower market value—about 2bn—as its cash equivalent. In the case of the Brady Plan resolution to the Latin American Debt Crisis of the 1980's, debt securities issued as part of the restructuring were also collateralized with US Treasury securities. While it might at first seem contradictory that a sovereign in default has access to cash, it is important to note that the analogy to domestic bankruptcy does not hold in the case of sovereign debt: Domestic bankrupts have limited liquid assets, whereas sovereigns typically have sufficient wealth to repay, but choose not to because of other spending priorities. For example, during the Latin American Debt Crisis of the 1980s, Mexico secretly purchased $8bn of its own

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[5] *Elliott Associates, L.P. v. Banco de la Nacion and The Republic of Peru*, 194 F.3d (2d Cir. 1999). See Alfaro 2006 for a summary.

[6] There is some debate as to whether credit market access was more easily regained during the latter half of the 1990s. See the debate between Gelos, Sahay and Sandleris (2004) and Richmond and Dias (2007).

debt on secondary markets (Cohen and Verdier 1995), while Peru repurchased \$1.7bn (Alfaro 2006)

The sovereign's inability to commit has manifested itself in another important way:

(IV) *Sovereign's cannot commit* ***not*** *to increase settlement payments over time.*

While a sovereign's typical catch-cry is never to concede to holdouts, the reality is different. There are numerous examples in addition to the Peruvian case cited above. In *CIBC Bank (Kenneth Dart) v. Brazil (1995)*[7], Dart purchased a deeply discounted second-hand debt with a face value of 1.4bn at discounts of greater than sixty per-cent. Dart refused to accept a Brady exchange, receiving 1.1bn—much more than other settling creditors. In *Elliott v. Republic of Panama (1997)*[8], Elliot received full payment while other creditors only managed a 50% discount. These successes are the primary motivation behind the rise in legal actions against sovereigns noted above.

(V) *Creditors incur substantial transactions costs, some of which are difficult to verify*

Evidence on the size of negotiation costs are available from a number of sources. Historically, non-profit bondholder organizations like the Corporation of Foreign Bondholders charged a fee of one-half of one per-cent of the value of the restructured debt in return for negotiating a restructuring agreement in order to defray costs. In the modern period, investment bank fees have sometimes exceeded this amount, with the Argentine "megaswap" of 2001 incurring fees of \$137 million on a \$30 billion bond restructuring, while the Russian restructuring of 1998 and the Mexican restructuring of 1984 both cost creditors 0.625% of their final settlements.[9] This presumably acts as a lower bound on costs to the extent that other costs incurred by creditors are not verifiable and cannot be shared as part of the restructuring agreement. For example, holdout creditors like Elliott Associated maintain a large staff and routinely complain about other creditors who free-ride on their legal expertise.[10]

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[7] *CIBC Bank and Trust Co (Cayman) Ltd v. Banco Central do Brazil, 886 F. Supp. 1105 (S.D.N.Y. 1995).*

[8] *Elliott Associates, L.P. v. Republic of Panama, 975 F. Supp. 332 (District, 1997).*

[9] For Argentina, see Blustein (2005); For the Mexican data, see Institute of International Finance (2001) *Survey of Debt Restructuring by Private Creditors*;for Russia, see *RussianFed 11% 98-18 24/01s ne (XS0089375249) - Prospectus,* filed with the Luxembourg Stock Exchange on July 7, 1998.

[10] "Elliott's activist chief has no time for cheats", *Financial Times*, 10th April 2006, p.4.

3 Illustrative Examples from the Basic Model

Below, our main results are presented using a series of simplified examples. For ease of exposition we adopt a fixed proportions bargaining protocol with two (or at most, three) creditors. We emphasize how the five salient features of the restructuring environment are incorporated into our model. Proofs, along with extensions to an arbitrary number of creditors, and to a wider class of bargaining protocols, are contained in subsequent sections.

Example 1: The 'Strategic Holdup Effect'

Here, we illustrate the strategic holdup effect and how it leads to delay. There are two identical creditors—which in practice are best thought of as creditor bargaining groups—and a sovereign debtor in default. Normal credit market access is worth V to the country; time is continuous; all parties discount the future at the same rate r, and information is complete. Adopting Feature II from above, each creditor can *individually* prevent the sovereign from realizing V by not engaging and consequently blocking the sovereign's normal access to international capital markets.

The creditors play a timing game of engagement and settlement which we solve by backwards induction: The last creditor to engage has a bargaining advantage because the first creditor is out of the picture, placing the last in a position of "bilateral monopoly" vis-a-vis the sovereign. The full gain V that the sovereign receives by re-establishing normal capital market access constitutes the bargaining 'pie'—we ignore any amount paid by the country to the previous creditor because, according to Feature III, it was an immediate cash payment that is now sunk. By Feature I, the sovereign cannot commit to a prior settlement and pays αV, so that the creditor receives

$$v_1 = \alpha V \left(1 - \phi\right), \tag{1}$$

where α is the fixed bargaining share, and ϕ is the proportion of surplus lost through transactions costs (Feature V). The subscript 1 on v_1 indicates that there is one creditor in the game. With only one creditor, there is no delay, so that the expected discount factor is $\beta_1 = 1$.

Moving back along the tree, consider the situation with two creditors, after one of them has entered. Both the sovereign and the first creditor to engage know that the second will receive αV before costs. Therefore, these two parties bargain over the anticipated net surplus $V - \alpha V$, and the first creditor receives

$$v_2 = \alpha \left(1-\alpha\right) V \left(1-\phi\right). \tag{2}$$

The subscript 2 on v_2 indicates that there are two creditors in the game and that the first to engage settles for v_2.

Moving back further in the game, suppose no creditor has engaged the sovereign. The *strategic holdup effect*[11] is delay caused by both creditors waiting before deciding to engage, each hoping to obtain the 'last-to-engage' payoff v_1 which is larger than the 'first-to-engage' payoff v_2.[12] Precisely, there is a symmetric Nash equilibrium in mixed strategies where both creditors choose the probability with which they will engage the sovereign in any instant of time, generating positive delay in expectation, with a corresponding expected cost of delay given by the interim discount factor $\beta_2 < 1$. The solution is characterized by an equilibrium probability density over engagement times for each player. Here, we simplify the analysis by noting that in a mixed strategy equilibrium, creditors must be indifferent between playing the mixed strategy, on one hand, and engaging immediately—on the other. Immediate engagement yields v_2, while playing the symmetric mixed strategy yields v_1 with probability 1/2 and v_2 with probability 1/2. It must be the case that in equilibrium the cost of delay sufficient makes creditors indifferent between these choices. Therefore β_2 satisfies

$$v_2 = \beta_2 \left[\frac{1}{2}v_1 + \frac{1}{2}v_2\right].$$

The cost of delay in the overall timing game with two creditors, denoted by the compound discount factor δ_2, is calculated as

$$\delta_2 \equiv \beta_2\beta_1 = \beta_2 = \frac{v_2}{\frac{1}{2}v_1 + \frac{1}{2}v_2}. \tag{3}$$

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[11] We borrow this terminology from the literature on the incomplete contracts theory of the firm which features holdup. See Grossman and Hart (1986; 1990), Williamson (1980) among many others.

[12] A higher payoff v_1 to the last creditor illustrates Feature IV, the sovereign's inability to commit *not* to increase settlements.

Substituting the values of v_1 and v_2 from (1) and (2) into this expression yields

$$\delta_2 = \frac{2\alpha (1-\alpha) V (1-\phi)}{\alpha V (1-\phi) + \alpha (1-\alpha) V (1-\phi)} = \frac{2(1-\alpha)}{2-\alpha}. \tag{4}$$

Three observations can be made immediately. First, since $\delta_2 < 1$ there is delay in equilibrium arising from the strategic holdup effect: The smaller is δ_2, the more the eventual payoffs are discounted, and the longer is delay. Second, delay is increasing in creditor bargaining power—an increase in α reduces δ_2. This is intuitive since the last creditor not only extracts more, but in doing so makes less is available to the first creditor. Third, expected delay is independent of both the size of the sovereign's gain V and the proportional negotiation cost ϕ, i.e. delay is homogenous degree zero in these magnitudes: It is the relative payoffs from the order of settlement that matter, not their absolute size. We establish the generality of these results in section 5. Further, we derive the effect of an arbitrary change in the payoff profile, showing that proportionate rises (falls) in payoffs as creditors settle lead to a rise (fall) in delay.

Example 2: Increasing the Number of Creditors

Recent years have seen a substantial rise in the number of creditors holding sovereign debt. To explore the effect of this on the dynamics of restructuring, this example extends the basic model of example 1 to three creditors—section 5.B considers an arbitrary finite number of creditors. With three creditors, the first to engage knows that the total current bargaining pie is limited by the amounts other creditors will extract in the future and is further diminished by an anticipated delay in future negotiations. Specifically, the total surplus is $(1-\alpha)^2 \delta_2 V$. It follows that the first creditor to engage receives the cash equivalent of

$$\alpha \left[(1-\alpha)^2 \delta_2 V (1-\phi) \right]. \tag{5}$$

We let $v_3 = \alpha (1-\alpha)^2 V (1-\phi)$ denote the undiscounted value of this payoff and note that $\delta_2 v_3$ is its discounted value.

The equilibrium interim discount factor, calculated similarly to the two player case, is

$$\beta_3 = \frac{v_3}{\frac{1}{3}v_1 + \frac{1}{3}v_2 + \frac{1}{3}v_3} < 1. \tag{6}$$

Therefore,

$$\delta_3 = \beta_3 \delta_2 < \delta_2, \tag{7}$$

and we can conclude that delay increases when another player is added. The underlying reason is that the lower payoff from being the first to settle increases competition for later payoffs. The general case of many players and arbitrary payoffs is significantly more difficult to derive as the addition of another player is viewed as an additional payoff at any point in the sequence of concessions (not just at the first point, as derived here). In section 5.B we derive reasonable conditions under which adding another player increases delay in the general model.

Example 3: CACs and Free Riding vs. Strategic Holdup

The problem of creditor holdout has led to a number of policy proposals aimed at reducing delay, ranging from the introduction of clauses into bond contracts that attempt to enforce collective action (Taylor 2002, 2006), through to the introduction of an international bankruptcy court (Krueger 2001, 2002(a),(b)). Collective action clauses have been widely adopted, e.g. being used in new bond issues in New York since Mexico's bond issue of 2003. However, the evidence on their effectiveness remains mixed. Here, we analyze the effect of these CACs and show that they may not reduce delay, even if they are successful at eliminating strategic holdup.

Consider the two creditor case but with a collective action clause that binds if fifty per-cent of creditors are in agreement. Creditors can no longer *individually* disrupt access to international credit markets: Rather, the agreement of the first creditor is sufficient to bind the second

To make it easier to compare the effect of the CAC with the uncoordinated negotiation of example 1, we assume that creditors' bargaining parameter is unchanged at α. Let φ denote uncontractible negotiation costs bourne by the creditor who negotiates under the CAC—in this case, with two players and a 50% rule, only one party negotiates in this way. We emphasize the possibility that the costs of negotiating within a collective may be different to 'individual' costs ϕ and, in particular, may be significantly smaller.

Suppose the first creditor reaches a settlement of v^c before costs, where the superscript c indicates the presence of a collective action clause. Since the last creditor is bound by the agreement, and since all creditors receive the same payment v^c, it must satisfy

$$v^c = \alpha \left(V - v^c \right).$$

In words, the first creditor to engage extracts a share α of $V - v^c$, the sovereign's value less the anticipated payout $v_1^c = v^c$ to the last creditor. The payoff before deducting negotiation costs is therefore

$$v^c = \frac{\alpha}{1+\alpha} V. \tag{8}$$

After incurring costs φ, the first creditor receives

$$v_2^c = \frac{\alpha}{1+\alpha} V \left(1 - \varphi \right). \tag{9}$$

The last creditor does not incur φ but is automatically limited to the agreement under the CAC, so that

$$v_1^c = v^c = \frac{\alpha}{1+\alpha} V.$$

Substitution of v_i^c into (3) yields discount factor

$$\delta_2^c = \frac{2\left(1-\varphi\right)}{2-\varphi},$$

which is decreasing in φ, so that expected delay is increasing in φ and bargaining power does not matter. Intuitively, the CAC removes competition otherwise present from bargaining. However, as the transactions costs of a collective action rise, the *free-rider motive* for holdout under collective action clauses is stronger.

Two conclusions can be made by comparing the cost of delay with CACs, δ_2^c, to the cost of delay without CACs, given by δ_2 from (4). First, whereas in the basic model delay depended on bargaining power and was independent of bargaining costs, with collective action clauses delay is independent of bargaining power and only depends on bargaining costs. Second, and in contrast to the views expressed in the policy literature, *collective action may induce greater delay than the 'uncoordinated' restructuring of the basic model.* In particular, if φ is high enough, the strategic-holdup motive is outweighed by the free-rider motive to hold-out. This is true even if collective action clauses reduce total transactions costs ($\varphi < \phi$): Relative (not absolute) transactions costs matter for delay.[13] Whether CACs generate less or greater delay is ultimately an empirical question.

In section 5.C we consider an arbitrary number of creditors N with a CAC that binds at an arbitrary threshold. In the general model, the key question becomes one of how creditors form a coalition of sufficient size to bring the clause into effect. We assume that creditors join the coalition by committing to an uncontractible share of group negotiation costs. The process of coalition formation itself is modeled as a timing game of entry to the coalition, with the first entrant being a 'founder'. We allow for the possibility that the first creditors to enter this coalition may have higher costs than the others. Creditors who never join avoid paying negotiation costs altogether, leading to a free rider incentive to hold out.

Example 4: Secondary Markets for Sovereign Debt

In the analysis above, we assumed that the number of creditors (which we can think of as the number of creditor bargaining groups) was fixed. In practice, the number and composition of creditors has changed with the development of secondary debt markets. In this example we allow debt to be traded among creditors in a competitive secondary market.

For simplicity, suppose we start with three creditors in the basic model and allow debt to be traded. For the sake of argument—and to establish a bound on the problem—we assume that the market is perfectly competitive and maximizes creditor value: Denoting π_i as the total market value with i creditors, we assume bonds are allocated to that number of creditors i solving $\max_i \pi_i$.[14] Rather than derive the exact conditions on α which determine

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[13] As emphasized by TAYLOR (2007), Chapter 4, the introduction of CACs into bond contracts has also been associated with the introduction of "engagement" and "initiation" clauses to streamline the restructuring process, and that will hopefully reduce the costs of negotiations.

[14] In making this assumption, we are ignoring the fact that the contracts necessary to achieve this outcome will have to overcome pecuniary externalities and could be quite complex. This issue is discussed later in

bond holdings here, we present the basic intuition and leave most of the derivations to section 5.D .

Taking the values of v_1, v_2, and v_3 from (1), (2) and (5), note that with three creditors the total value of the defaulted debt is $\pi_3 = 3\beta_2 v_3$: Each creditor can get $\beta_2 v_3$ by immediate concession, and all gains from competing for v_2 or v_3 are dissipated by delay. By similar logic, if debt holdings were consolidated among only two creditors, the total outstanding debt would have a value of $\pi_2 = 2v_2$, the discount factor being unity because the game ends as soon as the first creditor concedes. Consolidation to one creditor yields a total value of $\pi_1 = v_1$.

Starting with $i = 3$ and focusing on consolidation for simplicity (for now ignoring the possibility of dissolution to $i \geq 4$ creditors), the market equilibrium number of debt holders is

$$\max_i \left\{3\beta_2 v_3, 2v_2, v_1\right\}.$$

As bargaining power α increases, there are two effects, both of which serve to reduce creditor value. First, the strategic holdup effect is worsened, which reduces β_2 and hence tends to reduce $\pi_3 = 3\beta_2 v_3$. Second, since $v_3 \propto \alpha(1-\alpha)^2$, for high enough α, v_3 falls also, as there is less surplus for the early-engaging creditor to bargain over. Consequently, as α rises, consolidation first to two creditors, with $v_2 \propto \alpha(1-\alpha)$ and then to one creditor, with $v_1 \propto \alpha$ will be optimal. Thus, we have consolidation as α rises, and dissolution as α falls.

The effect of the emergence of secondary markets consequently depends on creditor bargaining power relative to the initial number of debt-holders. If α is high, and holdings are dispersed, then a secondary debt market will lead to consolidation, with an ambiguous net effect on delay. If α is low and holdings are concentrated, then the market will allocated debt to a greater number of creditors, again with an ambiguous net effect.

In Section 5.D we derive a general version of the result behind this analysis. We prove that there is a sequence of intervals $\cdots, [\alpha_j, \alpha_{j-1}], \cdots, [\alpha_3, \alpha_2], [\alpha_2, \alpha_1]$ with α_j strictly decreasing such that for bargaining power in that interval, consolidation to j creditors is optimal. We then apply the logic to the thought experiment of an emerging secondary debt market.

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section 5.D.

Example 5: ‘Vulture Creditors’

To this point, we have assumed that all creditors are identical and play symmetric strategies. Consequently, each creditor has an equal chance of holding out in equilibrium. In practice there is reason to believe that creditors are asymmetric: A lot of recent attention has been given the interaction between traditional and ‘vulture’ creditors, whose behavior differs from the norm, particularly in terms of its apparent willingness to hold out.

This example considers two types of vulture, both consistent with anecdotal evidence: One type—the legal entrepreneur—has specialized knowledge that allows it to extract a greater share of available surplus. The other type is simply more patient than the typical creditor. We then analyze the model with one vulture and one normal creditor to establish three results: First, the presence of a vulture with greater bargaining power increases overall delay. Second, delay is unaffected by the entrepreneurial vulture’s costs implying that delay rises even if the vulture incurs higher negotiation costs than a normal creditor. Third, if vulture creditors are more patient, delay is also increased.

Vultures as legal entrepreneurs

Let superscript V indicate the credit vulture’s, and superscript N indicate the normal creditor’s, parameters. We assume that vulture bargaining power is greater (indicating legal entrepreneurship) so that $\alpha^V > \alpha^N$, but the vulture’s transactions costs could have any relationship to the normal creditor’s costs, so that ϕ^V could be larger or smaller than ϕ^N. We are interested, in particular, in the possibility that vultures’ litigious tactics may be more expensive.

The backwards induction solution depends on the (randomly determined) order of settlement of the different kinds of creditor. If the vulture is last, it receives $a^V V\left(1-\phi^V\right)$. Moving backwards, the normal creditor anticipates that it will receive $\alpha^N\left(1-\alpha^V\right)V\left(1-\phi^N\right)$: This is because the vulture will extract a larger share $\alpha^V V$ of sovereign value when it settles. If, instead, the normal creditor is last, it receives $\alpha^N V\left(1-\phi^N\right)$ (as before) and the vulture anticipates receiving the surplus $\alpha^V\left(1-\alpha^N\right)V\left(1-\phi^V\right)$.

Mixed strategy equilibrium requires that each player be indifferent between immediate entry and playing a mixed strategy over engagement times. Let p denote the probability that the normal creditor (playing a mixed strategy) engages before the vulture, and let β_2 denote the expected discount factor corresponding to the delay before the first creditor engages. The indifference condition for the normal creditor is therefore

$$\alpha^N \left(1-\alpha^V\right) V \left(1-\phi^N\right) = \beta_2 \left[p\alpha^N \left(1-\alpha^V\right) V \left(1-\phi^N\right) + (1-p)\,\alpha^N V \left(1-\phi^N\right)\right]. \quad (10)$$

The LHS is the payoff from immediate entry. The RHS is the payoff to the normal creditor from mixing. Payoffs are discounted according to β_2, due to delay until the first creditor engages. With probability p, the normal creditor is first and receives $\alpha^N \left(1-\alpha^V\right) V \left(1-\phi^N\right)$, while with probability $(1-p)$ it is last to engage and gets $\alpha^N V \left(1-\phi^N\right)$. The indifference condition for the vulture is analogous:

$$\alpha^V \left(1-\alpha^N\right) V \left(1-\phi^V\right) = \beta_2 \left[(1-p)\,\alpha^V \left(1-\alpha^N\right) V \left(1-\phi^V\right) + p\alpha^V V \left(1-\phi^V\right)\right]. \quad (11)$$

Solving (10) and (11) for p and β_2 yields

$$p = \frac{\alpha^V \left(1-\alpha^N\right)}{\alpha^N + \alpha^V - 2\alpha^N\alpha^V}$$

and

$$\beta_2 = \frac{\alpha^N + \alpha^V - 2\alpha^N\alpha^V}{\alpha^N + \alpha^V - \alpha^N\alpha^V}.$$

First note that p is larger than one-half. This is because the vulture gets a relatively higher gain from delay than the normal creditor. Second, note that β_2 is independent of ϕ^v or ϕ^n, i.e. our conclusions hold regardless of whether or not the vulture faces higher bargaining costs than normal creditors. Third, note that the presence of the vulture increases delay, since $\alpha^v > \alpha^n$ and $p > 1/2$ implies

$$\beta_2 = \frac{1-\alpha^V}{(1-p) + p\left(1-\alpha^V\right)} < \frac{1-\alpha^N}{\frac{1}{2} + \frac{1}{2}\left(1-\alpha^N\right)} = \frac{2\left(1-\alpha^N\right)}{2-\alpha^N},$$

where the RHS is the expected discount factor with two normal creditors from example 1, equation (4).

The intuition for this result is as follows: Since the vulture's bargaining power is greater, there is less surplus available for the normal creditor should it engage the sovereign first. To deter the normal creditor from delaying, the vulture reduces the probability that it engages early, increasing the probability that the normal creditor concedes first, and increasing the cost of delay

The Vulture as a Patient Player

The assumption that all creditors have the same rate of time preference r has allowed us to ignore this variables in our calculations. Now suppose vulture creditors are more patient with discount rate r^V less than r^N. Now it is necessary to calculate expected delay instead of just focussing on discount factors. With details relegated to section 6 , we can show that

$$Et_2 = \frac{\alpha}{\left(\frac{r^V + r^N}{2}\right)(1-\alpha)},$$

i.e. expected delay Et_2 depends on the average discount rate of the creditors $\left(r^V + r^N\right)/2$. In particular, it increases the more patient is the vulture (i.e. as r^V falls). Starting with two normal creditors, if we make one of them more patient (increasing r^N to r^V for that creditor), then delay rises: Equilibrium requires the more patient vulture to maintain its indifference between immediate engagement and delay, and this entails greater delay in equilibrium.

4 The General Model

Above, simple examples were used to introduce our framework and exposit key findings. Some results are in line with received wisdom: Increasing debt dispersion by moving from two to three creditors will increase delay. Delay rises if one party is a vulture—either more patient or more 'entrepreneurial'. Other results, best thought of as instructive counter-examples, go against the standard intuition: CACs can increase delay if the free-rider effect outweighs the strategic holdup effect. There is greater debt-consolidation when creditor bargaining power is high. All of the examples assumed two or three creditors and assumed creditors received a fixed bargaining share. To ease exposition, we did not spend much time discussing the nature of the underlying game.

The current section has two main aims. One is to provide a rigorous game-theoretic foundation for the general model with an arbitrary finite number of players and payoffs (see the next two subsections). The other aim is to establish general results that encompass those in the example section. A surprisingly simple formula for expected delay is derived and shown to be quite general and very intuitive: With some restrictions on payoffs $v_N, v_{N-1}, \cdots, v_1$, expected delay is the ratio of the payoff from immediate concession to the mean payoff going forward, i.e.

$$\beta_i = \frac{v_i}{\frac{1}{i}\Sigma_{j=1}^{i} v_j}.$$

This is the analog of (3) for two players and (6), for three, extended to N players. We consider bargaining protocols other than the fixed-share protocol in section 3. We conduct comparative dynamics by allowing for arbitrary changes in the payoff profile $v_N, v_{N-1}, \cdots, v_1$ and by adding an additional creditor to N existing creditors. We model collective action arrangements with N creditors and a $100\gamma\%$ supermajority. The formation of collectives is endogenous, and is modeled a game of entry to a coalition that shares costs and behaves as one player. We extend the analysis of secondary debt markets with consolidation or dissolution of debt to an arbitrary finite number of players. In section 6 we study vulture creditors—by extending the model to one of asymmetric payoffs with M vultures and N normal creditors.

4.A Notation and Timing

There are N creditors and a sovereign. The game begins at time $t = 0$ when the sovereign fails to make debt payments. Each creditor then decides *when to engage* in a bilateral negotiation with the sovereign, exiting from negotiation by accepting a *settlement*. The settlement is a transfer of current resources—'cash', and cannot be renegotiated. The payment is taken to be exogenous. Sovereign behavior is introduced by proxy: We allow for explicit consideration of sovereign and creditor bargaining power by studying a range of different bargaining protocols and allowing arbitrary changes in the profile of settlements.

All creditors have complete information and their strategy is to choose an engagement time $t \in [0, \infty)$. If more than one creditor attempts to engage the sovereign at the same instant, one of them is selected at random. The game ends—and the sovereign regains undisrupted access to the international capital market—when all N creditors have accepted settlements.[15]

Our solution method is recursive, so it is helpful to adopt forward-looking notation. Let $i \in \{1, 2, \ldots, N\}$ index the number of creditors that remain in the game: This is the number who have **not** reached a settlement with the sovereign. The subgame in which there are i creditors who have not settled—correspondingly $N - i$ creditors who have settled—will be referred to as *subgame* i. It begins immediately with the $(N-i)^{th}$ settlement and continues until the N^{th} (or last) settlement. We derive symmetric, Markov-perfect Nash-equilibrium mixed strategies conditioned on the remaining number of creditors, i. The equilibrium is symmetric and in mixed strategies because we look for identical equilibrium CDFs over engagement times for all remaining players. It is Markov in that it is conditioned on the state variable i. Thus, strategies are captured by the CDF F_i with support being the time period between the (randomly determined) start of subgame i, to infinity. The strategies in the whole of subgame i are represented by the sequence $\{F_i, F_{i-1}, ..., F_2, F_1\}$. Thus, if there are i creditors playing according to F_i and one settles, the remaining group of $i - 1$ creditors plays according to F_{i-1} until another creditor settles, etc.

The cost of delay is captured by expected discount factors δ_i corresponding to each subgame $i = 1, ..., N$. Let β_i denote the interim expected discount factor for the period between the beginning of subgame i, at which time $N - i$ settlements have occurred, and the start of subgame $i - 1$, where $N - (i - 1)$ settlements have occurred. Thus we can define δ_i recursively as

$$\delta_i = \beta_i \delta_{i-1}, i = 2, ..., N. \tag{12}$$

If subgame i ends without delay then $\delta_i = 1$. If there is delay in expectation in subgame i, then $\delta_i < 1$. Figure 1, below, depicts expected timing of engagement, and the interim and full expected discount factors corresponding to such engagement.

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[15] This is without loss of generality. We can assume that the game ends when $\underline{N} < N$ creditors have signed. Among other possibilities this reflects the case where a critical mass of creditors, $\underline{N} < N$, is needed to hammer out a reorganization plan as with a collective action clause. By appropriate selection of payoffs, we can focus, without loss, on the case where $\underline{N} = N$.

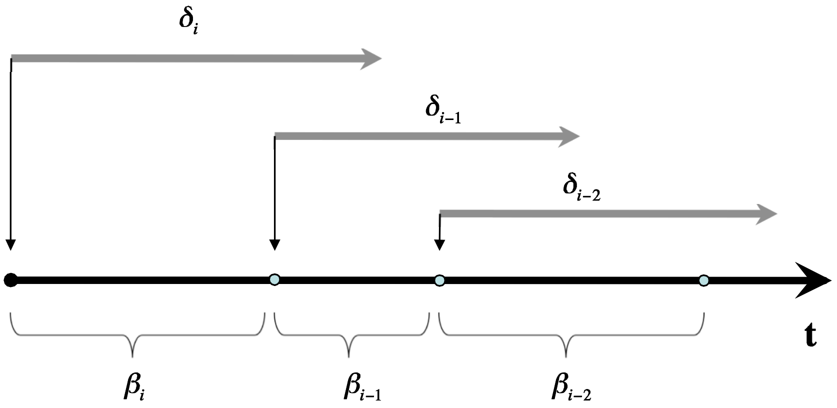

Figure 1: Timing

We allow for an arbitrary sequence of payoffs $v_N, v_{N-1}, ..., v_2, v_1$, where v_j is the payoff received by the creditor who is first to settle in subgame j. It turns out to be very convenient to define the v_i as the amounts received as at the instant of the *last* (N^{th}) settlement. The payoff as at the instant of first settlement in subgame i is denoted $u_i = \delta_{i-1} v_i$. The discount factor for the $(i-1)^{th}$ subgame is used, because this subgame begins immediately following the i^{th} settlement.

4.B Solution

By selecting an equilibrium in symmetric mixed strategies we can find a solution to the game as a sequence $F_N, F_{N-1}, ..., F_1$ of CDFs, where the F_i are the same for all players. We can view the timing game as a decision between engaging immediately and receiving u_i (if no others also try to engage) or waiting, with the chance of receiving the continuation payoff EU_{i-1} that is obtained by moving to a new game with one less creditor. The solution $F_N^*, F_{N-1}^*, ..., F_1^*$ is found assuming that both u_i and EU_{i-1} are fixed. We then solve the model recursively, starting with the last sugame and moving backwards until we reach the beginning of the game.

In subgame $i = 1$ there is only one player. Thus, there is no incentive to delay and the unique equilibrium has the creditor immediately settling with the sovereign. Normalizing the start of the subgame to zero, we have $F_1^*(0) = 1$, $\beta_1 = 1$ and $EU_1 = v_1$. For $i > 1$, the solution depends on the magnitudes of u_i and EU_{i-1}.

If $u_i \geq EU_{i-1}$, then $F_i^*(0) = 1$, and $\beta_i = 1$: Players prefer to engage immediately and get the prize u_i. Since more than one creditor attempts to engage the sovereign in this case, we adopt the tie-breaking rule that creditors face equal probabilities of being first, i.e.

$$EU_i = \frac{1}{i} u_i + \frac{i-1}{i} EU_{i-1}.$$

If $u_i < EU_{i-1}$, then each creditor has an incentive to delay in the hope of getting EU_{i-1}. The equilibrium F_i is computed by noting that if all others randomize according to F_i, a creditor is indifferent between each pure strategy engagement decision. In particular a creditor is indifferent between immediate engagement, in which case EU_i is received, and engagement T units of time after the beginning of the subgame. The latter payoff depends on the choices of the $i-1$ other creditors. Denote τ_{m} as the minimum engagement time of the other creditors, and suppose it has density g_{i-1}. The indifference condition can be written as

$$u_i = u_i e^{-rT} [1 - F_i(T)]^{i-1} + EU_{i-1} \int_0^T e^{-r\tau_{\mathrm{m}}} g_{i-1}(\tau_{\mathrm{m}}) d\tau_{\mathrm{m}} \tag{13}$$

The term $u_i e^{-rT} [1 - F_i(T)]^{i-1}$ is the expected payoff if no other creditors engage before T (which they do with probability $[1 - F_i(T)]^{i-1}$). The expression

$$\int_0^T e^{-r\tau_{\mathrm{m}}} g_{i-1}(\tau_{\mathrm{m}}) d\tau_{\mathrm{m}}$$

is the expected discount factor from the minimum of the $i-1$ other creditors' random engagement decisions, and EU_{i-1} is the payoff if $\tau_{\mathrm{m}} < T$. The solution method is to exploit the fact that the creditor is indifferent between entering at T and entering at $T + dT$, by differentiating (13), setting the result to zero, and solving the resulting differential equation. The complete solution for all values of u_i and EU_{i-1} along with proof is given in Lemma 8 in the appendix section 8.A .

If $u_i < EU_{i-1}$ the solution is

$$F_i^*(t) = 1 - \exp\left\{ -\frac{r u_i}{(i-1)\left[u_{i-1} - u_i\right]} t \right\}.$$

Equilibrium entails (random) delay in which all gains are exhausted in expectation. This provides us with a simple way of interpreting the interim expected discount factor β_i (we characterize β_i formally in the appendix). Players are indifferent between receiving u_i immediately, or mixing and getting u_i with probability $1/i$ and EU_{i-1} with probability $(i-1)/i$: The cost of delay β_i must equate these payoffs, i.e.

$$\beta_i = \frac{u_i}{\frac{1}{i}u_i + \frac{i-1}{i}EU_{i-1}}.$$

Given an arbitrary sequence of payoffs v_i, for $i = 1, ..., N$, and noting that $u_i = \delta_{i-1}v_i$, recursive application of the two-payoff game (with fixed u_i and EU_{i-1}) allows us to characterize the entire equilibrium strategy profile:

Proposition 1. *Consider a sequence of strictly positive payoffs* $v_N, v_{N-1}, ..., v_1$. *The expected payoff for subgame* i, *expected discount factor for the time from the start of the subgame until the* $N-(i-1)^{th}$ *concession, and the equilibrium CDF of concession times are determined recursively, starting at* $i = 2$ *with* $\delta_1 = \beta_1 = 1$ *and* $EU_1 = u_1 = v_1$ *as follows: Set* $u_i = \delta_{i-1}v_i$ *and calculate*

$$EU_i = \min\left\{u_i, \frac{1}{i}u_i + \frac{i-1}{i}EU_{i-1}\right\}, \tag{14}$$

$$\beta_i = \frac{iEU_i}{(i-1)EU_{i-1} + u_i}, \tag{15}$$

$$F_i^*(t) = \left\{ \begin{array}{cc} 1 - \exp\left\{-\frac{ru_i}{(i-1)[EU_{i-1}-u_i]}t\right\} & \text{for } u_i < EU_{i-1},\ t \in [0,\infty) \\ 1 & \text{for } EU_i \geq EU_{i-1},\ t = 0 \end{array} \right., \tag{16}$$

and

$$\delta_i = \beta_i \delta_{i-1}. \tag{17}$$

Repeat for $i \to i+1$, *and stop at* $i = N$.

Proof. See the appendix section 8.B. □

This proposition gives the solution to the game for any sequence of positive payoffs, increasing, decreasing or non-monotonic. Therefore it allows for cascades of engagement as well as delay as the following example shows:

Example 1. *With the three player game and sequence* $v_3 > v_2 > v_1$*, we calculate* $u_2 = v_2$, $EU_2 = v_2/2 + v_1/2$ *and* $\beta_2 = 1$*, and;* $u_3 = v_3, EU_3 = v_3/3 + v_2/3 + v_1/3$ *and* $\beta_3 = 1$*. There is no delay in either subgame as all creditors prefer the higher prizes with winners are chosen at random. If* $v_3 < v_2 < v_1$*, then we calculate* $u_2 = EU_2 = v_2$*, and* $\beta_2 = 2v_2/\left(v_1 + v_2\right)$*, and;* $u_3 = EU_3 = v_3$ *and* $\beta_3 = 3v_3/\left(v_1 + v_2 + v_3\right)$*. There is delay in both subgames as players exhaust the gains from seeking higher payoffs from later engagement. Even if payoffs are not decreasing as we move into later subgames, there may be no delay in equilibrium. Suppose that* $v_3 < v_2$ *and* $v_2 > v_1$*. Then* $u_2 = v_2$, $EU_2 = v_2/2 + v_1/2$ *and* $\beta_2 = 1$*, and;* $u_3 = v_3$, $EU_3 = v_3$ *for* $v_3 < v_2/2 + v_1/2$ *and* $\beta_3 = 3v_3/\left(v_1 + v_2 + v_3\right) < 1$ *or* $EU_3 = v_3/3 + v_2/3 + v_1/3$ *for* $v_3 > v_2/2 + v_1/2$ *and* $\beta_3 = 1$*.*

Proposition 1 is presented for completeness, specifically (as mentioned) to show that the model is capable of exhibiting cascades of engagement as well as delay along the equilibrium path. Since we are mainly concerned with delay—and how it is affected by changes in payoff profiles—we will make assumptions that ensure delay occurs in-between each settlement (except between the final two). This happens, for example, if the sequence of payoffs $v_1, \cdots, v_{N-1}, v_N$ is monotonically weakly decreasing in i meaning that later settlers receive weakly higher payoffs. As is suggested by the non-monotonic example above which shows that

$$v_3 < \frac{1}{2}v_2 + \frac{1}{2}v_1$$

is sufficient for delay even though v_2 is greater than v_1, this weak monotonicity is stronger than we need. Let

$$\mu_i = \frac{1}{i}\sum\nolimits_{j=1}^{i} v_j,$$

denote the average payoff for all settlements from 1 to i in the following proposition:

Proposition 2. *Consider a sequence of non-negative payoffs* $v_N, v_{N-1}, ..., v_1$*. If*

$$v_i < \mu_i \text{ for } i = 2, ..., N \tag{18}$$

then there is delay in making all concessions except the last, with expected discount factor

$$\beta_i = \frac{v_i}{\mu_i}, \tag{19}$$

expected time until the first concession in subgame i

$$Et_i = \frac{i-1}{i} \frac{\mu_{i-1} - v_i}{r v_i}, \tag{20}$$

where t_i is distributed exponentially with parameter $[Et_i]^{-1}$, and expected payoff

$$EU_i = \delta_{i-1} v_i. \tag{21}$$

Proof. See the appendix section 8.C. □

This result demonstrates that the interim expected discount factor (19) is the ratio of the gain from immediate settlement to the payoff from the gamble of waiting and being allocated any of the other settlements with equal probability (due to symmetric mixing). The example immediately following proposition 1 with $v_3 < v_2$ and $v_2 > v_1$ shows that condition (18) in proposition 2 is less restrictive than weak monotonicity.[16]

4.C Alternative Bargaining Protocols

The general model has so far featured an arbitrary profile of settlements $v_N, v_{N-1}, ..., v_1$. In this subsection, we discuss some bargaining protocols that may determine the v_i. While much of the focus of our examples will be on the constant bargaining share case—which is a convenient leading example that yields closed form solutions in much of the analysis below—we present alternative bargaining protocols to demonstrate the flexibility of our approach.

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[16] It is also worth noting that the sufficient condition $v_i < \mu_i$ for all $i \geq 2$ corresponds to the condition that the sequence of expected values of each settlement, each valued at the time of settlement, which we have been denoting by u_i, is monotonically increasing as the number of creditors remaining to settle falls, or $u_i < u_{i-1}$ for all $i \geq 2$. This follows since

$$u_i = \delta_{i-1} v_i = \beta_{i-1} \delta_{i-2} v_i = \frac{v_{i-1}}{\mu_{i-1}} \delta_{i-2} v_i < \delta_{i-2} v_{i-1} = u_{i-1},$$

where we made use of the fact that $v_i < \mu_{i-1}$ as shown in the previous paragraph.

In the examples below we work with the surplus

$$V - \sum_{k=1}^{i-1} v_k^g, \tag{22}$$

which is the value, measured at the time the last creditor settles, of the net surplus available in the bargain between a creditor and the sovereign in subgame i. It consists of V, the value of undisrupted credit market access, less anticipated payments to future-settling creditors. The superscript g on v_k^g indicates that the surpluses are gross of any transactions costs which the creditors bear individually. The expression (22) is important because it is directly proportional to the *current* value of the 'pie' over which parties will bargain, measured at the time of settlement in subgame i.

We will use the following bargaining protocols in various examples throughout the remainder of the paper.

Constant Bargaining Share

The simplest protocol has each creditor receive a share α of available surplus, i.e.

$$v_i = \alpha \left[V - \sum_{k=1}^{i-1} v_k^g \right]. \tag{23}$$

Nash Bargaining

Generalized Nash bargaining has parties receive their threat-point payoff, which we denote l for creditors and L for the sovereign, plus a share of the gains from negotiation. One interpretation of the threat-points has l as the gain to a creditor from litigation, and $-L > 0$ as the cost of the sovereign's defense. Under this protocol, the i^{th} creditor to settle receives

$$v_i = l + \alpha \left(V - \sum_{k=1}^{i-1} v_k^g - l - L \right). \tag{24}$$

Bargaining with an Outside Option

So far we have assumed that the value any single creditor can extract is independent of the size of its holding. In practice, the sovereign always has the option of repaying the debt in full. To capture this possibility, we consider a protocol under which the sovereign has the 'outside option' to settle in full. Let the face value of the debt held by each creditor be $b = B/N$, where B is the total current value of debt and N is the number of creditors. Assume (as was the case until 1997) that courts do not award deferred interest, so that the outside option is fixed in current value at b. In our framework, this protocol yields payoffs[17]

$$v_i = \min\left\{\alpha\left(V - \sum_{k=1}^{i-1} v_k^g\right), b\right\}. \tag{25}$$

Intuitively, a creditor can extract the share α of remaining surplus as long as this yields an amount less than b.

5 Results and Applications

Here we use the solution given by proposition 2 to show that the results in the example section generalize. The comparative static and comparative dynamic results are of independent interest as properties of timing games.

Basic Model (Constant Bargaining Share) The basic model of the example section adopts a constant bargaining share protocol. It is straightforward to generalize this to the case of many creditors. In the two-creditor case $v_1^g = \alpha V$ and $v_2^g = \alpha(1-\alpha)V$. Therefore, using (22) we have

$$\begin{aligned} v_3^g &= \alpha[V - v_1^g - v_2^g] \\ &= \alpha[V - \alpha V - \alpha(1-\alpha)V] \\ &= \alpha(1-\alpha)^2 V, \end{aligned}$$

so that the payoff net of negotiation costs is

$$v_3 = \alpha(1-\alpha)^2 V(1-\phi).$$

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[17] See BINMORE ET AL (1986).

By iteration we have

$$v_i = \alpha \left(1-\alpha\right)^{i-1} V \left(1-\phi\right), \tag{26}$$

which is decreasing in i. Using proposition 2, substitution of (26) yields the interim discount factor

$$\beta_i = \frac{i\left(1-\alpha\right)^{i-1}}{\sum_{k=1}^{i}\left(1-\alpha\right)^{k-1}}. \tag{27}$$

The strategic holdup effect generalizes to many creditors under the constant-bargaining-share protocol: Delay occurs as creditors seek to take advantage of the larger settlements available in later subgames—since by (26) v_i is decreasing in i. Moreover, we can see by examination of the interim discount factor (27) that delay is determined (for a given rate of time preference) entirely by the number of creditors present in the subgame and their bargaining share. Delay is independent of the country's value of undisrupted international financial market access, and of proportional negotiation costs. In section 5.A below we generalize the comparative statics results from our examples section, i.e. the effect of; changes in bargaining power, changes in the number of players, CACs, the introduction of vulture creditors, and the impact of markets for second-hand bonds.

Nash Bargaining The constant share protocol is a special case of Nash bargaining, with threat-point payoffs set equal to zero. Similarly to the above, the last creditor to settle gets

$$v_1 = l + \alpha\left(V_L - l\right).$$

where $V_L \equiv V - L$. The second receives

$$\begin{aligned} v_2 &= l + \alpha(V_L - l - v_1) \\ &= l + \alpha(V_L - l - [l + \alpha\left(V_L - l\right)]) \\ &= \alpha\left(1-\alpha\right)V_L + l\left(1-\alpha\right)^2, \end{aligned}$$

and by iteration the first settlement in subgame i is

$$v_i = \alpha \left(1-\alpha\right)^{i-1} V_L + \left(1-\alpha\right)^i l. \tag{28}$$

Direct substitution of (28) into (19) yields

$$\beta_i = \frac{\alpha \left(1-\alpha\right)^{i-1} V_L + \left(1-\alpha\right)^i l}{\frac{1}{i}\sum_{k=1}^{i} \alpha \left(1-\alpha\right)^{k-1} V_L + \left(1-\alpha\right)^k l}, \; i = 1, ..., N.$$

The strategic holdup effect is still present as v_i is decreasing in i. Perhaps surprisingly, it turns out that delay is still independent of country size, as we shall see below.

Outside Options Bargaining Protocol In this case delay depends on how much can be extracted in a bargain, $\alpha\left(V - \Sigma v_k^g\right)$, compared with the face value of an outstanding claim on the country's debt, b. For example, if $b \geq \alpha V$, then $v_1 = \min\left\{\alpha V, b\right\} = \alpha V$. That is, the last creditor extracts less than the face value of the debt. Since $\alpha\left(1-\alpha\right)^{i-1} V < \alpha V \leq b$ for all $i \geq 1$, equation (25) yields $v_i = \alpha\left(1-\alpha\right)^{i-1} V$, so that the payoff profile is the same as in the constant bargaining share case above.

If $b < \alpha V$, the sovereign pays the last creditor b, and, iterating backwards, will pay others b unless it becomes cheaper to bargain. Consider equation (25) and suppose there is an integer i^* such that

$$v_{i^*-1} = \min\left\{\alpha(V - (i^* - 2)b), b\right\} = b \tag{29}$$

and

$$v_{i^*} = \min\left\{\alpha(V - (i^* - 1)\, b), b\right\} = \alpha(V - (i^* - 1)\, b). \tag{30}$$

In words, the (first) bargaining payoff in subgame $i^* - 1$ exceeds the face value, but the bargaining payoff in subgame i^* is less than face value.[18]

If (29) and (30) hold, it is better to settle in full in all subgames $i < i^*$, and to bargain in *all* subgames $i \geq i^*$, since the bargaining payoff is decreasing. Thus, the payoff profile is

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[18] Such an i^* exists as a result of the monotonicity in i^*, of the bargaining term $\alpha(V - (i^* - 1)\, b$ in (30). However if $i^* > N$, then all creditors are repaid in full.

given by

$$v_i = \begin{cases} b & \text{for } i < i^* \\ \alpha \left(1-\alpha\right)^{i-i^*} \left(V - \left(i^*-1\right) b\right) & \text{for } i \geq i^* \end{cases} . \tag{31}$$

The discount factors corresponding to this profile are

$$\beta_i = \begin{cases} 1 & \text{for } i < i^* \\ \frac{i^* \alpha (V-(i^*-1)b)}{\alpha(V-(i^*-1)b)+(i^*-1)b} & \text{for } i = i^* \\ \frac{i(1-\alpha)}{(1-\alpha)+(i-1)} & \text{for } i > i^* \end{cases} .$$

Note that they depend on the face value of debt and the value of market re-access to the country. With outside-options bargaining, delay does depend on country "size" V (relative to individual indebtedness b), contrary to the case for the other bargaining protocols.

5.A Changes in Settlement Profiles

Here we characterize the effect of changes in $v_N, v_{N-1}, ..., v_1$, and apply the results to the bargaining protocols above. First, note that delay is unaffected by a proportionate change in all payoffs from v_j to γv_j, $\gamma > 0$.

Remark 1. *Expected discount factors β_i, are homogeneous of degree zero in the payoff profile.*

Creditors' equilibrium behavior is unaffected by proportionate changes in the value of debt. An immediate implication is that the scale of borrowing has no impact: other institutional factors unchanged, small and large debtor countries experience identical delay in restructuring.

More generally, we are interested in how delay is affected by an arbitrary change the payoff profile. To parameterize the problem, define $v_k = v_k(s)$, $k = 1, ..., N$. By (19), $\log \beta_j(s) = \log v_j(s) + \log \mu_j(s)$. Differentiating with respect to s yields

$$\widehat{\beta}_j = \widehat{v}_j - \frac{\Sigma_{k=1}^{j} \theta_k \widehat{v}_k}{j}, \tag{32}$$

where hats represent derivatives of logs, and $\theta_k \equiv v_k / \mu_j$. Equation (32) is straightforward to interpret: if s causes the immediate payoff to rise ($\widehat{v}_j > 0$), this tends reduce delay, but if it causes the average of payoffs going forward to rise $\left(\Sigma_{k=1}^{j} \theta_k \widehat{v}_k / j > 0\right)$, this tends to increase delay. The net effect is the difference given by the RHS of (32). Creditors care about the change in what they could gain immediately relative to the change in the payoff from waiting. Note for completeness that the change in expected discount factors for any subgame is found by differentiating the log of $\delta_i = \Pi_{j=1}^{i}\beta_j$ and substituting (32) to get

$$\widehat{\delta}_i = \sum_{j=1}^{i}\left[\widehat{v}_j - \frac{\Sigma_{k=1}^{j}\theta_k\widehat{v}_k}{j}\right].$$

Several comments are in order. We can confirm the homogeneity result using (32) by noting that if $\widehat{v}_k = \widehat{v}$ for $k \leq j$, then $\widehat{\beta}_j = 0$. Further, it is immediate that if payoffs rise more than proportionately as k falls—that is if $\widehat{v}_k$ is rising as k falls for $k \leq j$—then $\widehat{\beta}_j < 0$. Similarly if $\widehat{v}_k$ falls as k rises for $k \leq j$ then $\widehat{\beta}_j > 0$. The following examples demonstrate the utility of this result.

Example 2 (Basic model, constant share, many players)**.** *Consider the constant-share protocol. Letting* $s = \alpha$*, differentiation of* $\log v_j$ *from (26) yields*

$$\widehat{v}_j = \frac{\partial \log v_j}{\partial \alpha} = \frac{(1 - j\alpha)}{\alpha(1-\alpha)}.$$

Since this is decreasing in j*,* β_j *is decreasing. Substituting this into (32) and rearranging gives*

$$\hat{\beta}_i \propto \sum_{j=1}^{i}\alpha(j-i)\,v_j < 0 \text{ for } i > 1.$$

In the generalized basic model, a rise in creditor bargaining power reduces each interim discount factor and so increases expected delay in any subgame.

Example 3 (Nash bargaining)**.** *For* $s = \alpha$*,*

$$\frac{\partial \log v_i}{\partial \alpha} = \frac{(1-i\alpha)\,V_L - i\,(1-\alpha)\,l}{\alpha(1-\alpha)\,V_L + (1-\alpha)^2\,l} < 0,$$

so that $\hat{\beta}_i < 0$ *and* $\widehat{\delta}_i < 0$ *for all* i*. For* $s = l$*,*

$$\frac{\partial \log v_i}{\partial l} = \frac{(1-\alpha)}{\alpha V_L + (1-\alpha)\,l},$$

which is independent of i*, so that* $\beta_i = 0$ *and* $\widehat{\delta}_i = 0$*. Changes in litigation settlements do not affect delay. For* $s = V_L$*,*

$$\frac{\partial \log v_i}{\partial V_L} = \frac{\alpha}{\alpha V_L + (1-\alpha)\, l}$$

is also independent of i*, so that changes in country size do not affect delay.*

Example 4 (Bargaining with an option to repay the face value)**.** *For this bargaining protocol, overall delay may either rise or fall with bargaining power* α*. To see this, we apply the formula (32) to the expression (31) for the payoff profile. In later subgames where all creditors are paid in full (i.e. for* $i < i^*$*),* $\hat{\beta}_i = 0$*, since the face value* b *is independent of* α*. In later subgames, where all creditors bargain (i.e. for* $i > i^*$*), relative payoffs are the same as for the constant share bargaining protocol, and delay increases as before. For subgame* i^**, delay is decreasing because the payoff from bargaining is rising relative to the payoff from settlement in full. Hence, the effect on overall delay is ambiguous.*

5.B Changing the Number of Creditors

The last decade or so saw a move away from bank loans to bonds and a substantial rise in participation in the market for sovereign debt. This led to concern that settlements would become more protracted. Here, we use the general model to investigate the effect on the discount factor of a rise in the number of participating creditors.

A good way to think about the problem of increasing N is to consider the game as we have analyzed it with payoff sequence

$$\mathbf{v} = (v_N, v_{N-1}, \cdots, v_j, v_{j-1}, \cdots, v_2, v_1)\,, \tag{33}$$

and to compare it to the game with one extra player along with one additional payoff. The new game will have payoff sequence $\mathbf{v}'$ which depends on where the extra payoff is placed. If we add an extra player by keeping all other payoffs the same in $\mathbf{v}$ and placing a new payoff v'_j in-between v_j and v_{j-1} for $j : N \geq j > 1$ then

$$\mathbf{v}' = \left(v_N, v_{N-1}, \cdots, v_j, v'_j, v_{j-1}, \cdots, v_2, v_1\right). \tag{34}$$

If we add a new player along with a payoff at the beginning of the game we have

$$\mathbf{v}' = (v'_{N+1}, \mathbf{v}), \tag{35}$$

and if we add another player along with a payoff at the end of the game then

$$\mathbf{v}' = (\mathbf{v}, v'_1). \tag{36}$$

(I) *Adding a payoff to the beginning*

An example of this is the generalized basic model with proportional bargaining, where adding another player adds the payoff

$$v'_{N+1} = \alpha (1-\alpha)^N V (1-\phi)$$

in (35). More generally, we will add a payoff v'_{N+1} which satisfies our maintained assumption

$$v'_i < \mu'_i, \text{ for all } i \geq 2. \tag{37}$$

This case is easy to analyze, because the effect of adding another creditor is identical to the question of how, for any sequence of payoffs satisfying (37), δ_i changes as i increases. From the definition $\delta_{i+1} = \beta_{i+1}\delta_i$ and with $\beta_{i+1} < 1$ by (37), it is clear that $\delta'_{N+1} < \delta_N$. The effect of adding another player and a new payoff for the first concession, is a lower expected discount factor and increased expected delay.

(II) *Adding a payoff to the end*

When changing the number of creditors affects the settlement outcomes that other creditors can attain—as is the case when a payoff is added to the end of the game—the problem is more complicated. We seek to compare the expected discount factor with payoffs $\mathbf{v}$ and N creditors to that from $\mathbf{v}' = (\mathbf{v}, v'_1)$ and $N+1$ creditors, i.e. to find δ'_{N+1}/δ_N. Note that

$$\frac{\delta'_{N+1}}{\delta_N} = \frac{\beta'_{N+1} \cdots \beta'_3 \beta'_2 \beta'_1}{\beta_N \cdots \beta_2 \beta_1} = \left(\frac{\beta'_{N+1}}{\beta_N}\right) \cdots \left(\frac{\beta'_3}{\beta_2}\right) \left(\frac{\beta'_2}{\beta_1}\right).$$

since $\beta'_1 = 1$, so that we can make the comparison by calculating the generic term β'_{i+1}/β_i. In (36) we have

$$v'_{N+1} = v_N, v'_N = v_{N-1}, \cdots, v'_{i+1} = v_i, \cdots, v'_2 = v.$$

Therefore

$$\frac{\beta'_{i+1}}{\beta_i} = \frac{v'_{i+1}/\mu'_{i+1}}{v_i/\mu_i} = \frac{\mu_i}{\mu'_{i+1}} = \frac{\frac{1}{i}(v_1 + \cdots + v_i)}{\frac{1}{i+1}(v'_1 + v_1 + \cdots + v_i)} < 1$$

since $v'_1 > v_1$: Expected delay increases.

(III) *Adding a payoff to the middle*

The following proposition compares the outcomes from the game with an additional player and an extra payoff strictly in the middle of the sequence $\mathbf{v}$.

Proposition 3. *Suppose the game with payoffs (33) is replaced with the game under (34) with the extra payoff v'_j in-between v_j and v_{j-1} and that $v_i < \mu_i$ for $i = 1, ..., N$, and $v'_i < \mu'_i$ for $i = 1, ..., N+1$.*

(a) *If $v_j \leq v'_j$, then the expected delay in the realization of the settlement in the j^{th} subgame in the original game is less than the expected delay in the realizations of both $v'_{j+1} = v_j$ and v'_j in the new game, equivalently $\beta_j > \beta'_{j+1}\beta'_j$, and;*

(b) *if $v'_j > \mu_i$ for all $i \geq j+1$, then $\beta_i > \beta'_{i+1}$ for $i = j+1, ..., N$, and if in addition $v'_j \geq v_j$, the new game has greater expected delay, equivalently $\delta'_{N+1} < \delta_N$.*

Proof. For part (a) note that

$$\frac{\beta_j}{\beta'_{j+1}\beta'_j} = \frac{jv_j}{v_1 + ... + v_j} \frac{(v_1 + ... + v'_j + v_j)(v_1 + ... + v'_j)}{(j+1)jv_jv'_j}$$

which is greater than unity if Δ (defined below) is positive, i.e.

$$\begin{aligned}
\Delta &= (v_1 + ... + v'_j + v_j)(v_1 + ... + v'_j) - (j+1)v'_j(v_1 + ... + v_j) \\
&= (v_1 + ... + v_j)\left[(v_1 + ... + v'_j) - jv'_j\right] + v'_j\left[(v_1 + ... + v'_j) - (v_1 + ... + v_j)\right] \\
&= j(v_1 + ... + v_j)(\mu'_j - v'_j) + v'_j(v'_j - v_j) > 0.
\end{aligned}$$

The inequality follows since $\mu'_j > v'_j$ and $v'_j \geq v_j$.

For part (b) note that for $i \geq j+1$,

$$\frac{\beta_i}{\beta'_{i+1}} = \frac{i(v_1 + \ldots + v'_j + \ldots + v_i)}{(i+1)(v_1 + \ldots + v_i)},$$

which is no less than unity if

$$\begin{aligned} \Delta^* &= i(v_1 + \ldots + v'_j + \ldots + v_i) - (i+1)(v_1 + \ldots + v_i) \\ &= iv'_j - (v_1 + \ldots + v_i) \end{aligned}$$

is non-negative. If $v'_j \geq \mu_i, i \geq j+1$,then $\Delta^* \geq 0$ as required.

Finally, since $\delta'_{N+1} = \prod_{i=1}^{N} \beta'_{i+1}$ and $\delta_N = \prod_{i=1}^{N} \beta_i$ we have $\delta'_{N+1} < \delta_N$. $\square$

The conditions of the proposition require that the payoff that is added is sufficiently large. If $v'_j \geq \mu_i, i \geq j+1$ then a player is more likely to receive the higher payoff v'_j if she reduces the likelihood of claiming any concession $i \geq j+1$.

Corollary 4. *If payoffs are strictly monotonic, and* $v'_j \geq \mu_{j+1}$*, then adding a creditor along with a payoff in any position leads to increased expected delay.*

Proof. Strict monotonicity and $v'_j \geq \mu_{j+1}$ ensures that $v'_j > v_j$, and that $v'_j \geq \mu_{j+1} > \mu_{j+2} > \cdots > \mu_N$. $\square$

To summarize, there are reasonable conditions under which adding another player along with another payoff leads to greater delay, regardless of the position in which the new payoff appears.

Below, we analyze the effect of vulture creditors who are either entrepreneurial or more patient than normal creditors. Another possibility is that vulture creditors are players who stubbornly hold out for the highest payoff regardless of the behavior of others. The impact of such a stubborn vulture is equivalent to moving from a game with $N+1$ normal creditors to one with N normal creditors and a vulture that always gets v'_1, i.e. from profile $(\mathbf{v}, v'_1)$ to profile $\mathbf{v}$. We showed above that delay is lower with fewer creditors: the impact of a stubborn vulture is to reduce delay! What happens here is that the vulture improves matters by eliminating competition between other creditors for the highest payoff v'_1. Of course, if there were two such (irrationally) stubborn vultures, delay would be infinite.

5.C Collective Action Mechanisms

Historically, creditors formed bondholder committees—such as the British *Corporation of Foreign Bondholders*—to negotiate with sovereigns in default. Recently, bond contracts have included collective action clauses that permit a supermajority of creditors to impose a settlement on holdouts. In this section, we model a general 'collective action mechanism' (encompassing CACs and bondholder committees), with two key features: First, the mechanism has all creditors receive the same settlement, gross of negotiation costs. Second, creditors who engage the sovereign can only bargain collectively (i.e. can only bargain as if they were one party). A collective bargain happens only once enough creditors agree to bear the individual costs ϕ, assumed to be non-verifiable, due to participating in the collective action.[19] Despite the fact that all creditors are paid the same settlement, not all creditors get the same payoff net of costs, because those who join the collective action pay ϕ.

To calculate the settlement, let M denote the minimum number of creditors needed to satisfy a given $100\gamma\%$ supermajority rule. That is, M satisfies

$$\frac{M}{N} \geq \gamma > \frac{M-1}{N}.$$

Let v^{cN} denote the common settlement under the collective action mechanism (where superscript cN indicates 'collective action with N creditors'). As there are M creditors who join the collective action, their total payoff will be Mv^{cN}. Since they bargain as a collective, i.e. as one party, they receive a fraction α of the anticipated future surplus. The latter is equal to $V-(N-M)\,v^{cN}$, since there are $N-M$ creditors who do not join the action, each receiving v^{cN}. Thus,

$$Mv^{cN} = \alpha[V-(N-M)\,v^{cN}].$$

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[19] Acceptance of such costs is reflected, for example, in the decision to join a council or bondholder committee (as in the case of the Global Committee of Argentine Bondholders which charged membership fees) or in tendering bonds as part of a debt restructuring that used exit consents, or with the invocation of a collective action clause.

Re-arranging yields the individual payment before costs as

$$v^{cN} = \frac{\alpha}{M + \alpha\left(N - M\right)} V \equiv \alpha^{*} V.$$

Therefore, the payoff profile under a collective action mechanism is

$$v_i^{cN} = \begin{cases} \alpha^{*} V \left(1 - \phi\right) \text{ for } i = N, ..., N - M \\ \\ \alpha^{*} V \text{ for } i = N - M - 1, ..., 1 \end{cases}. \tag{38}$$

The first M creditors join the coalition and each pays ϕ, and the remaining $N - M$ free-ride. This leads to the following result:

Proposition 5. *A collective action mechanism yields delay corresponding to the expected discount factors*

$$\beta_i = \begin{cases} \frac{(N-M+1)(1-\phi)}{(N-M-1)\phi + i(1-\phi)} & \textit{for } i = N, ..., N - M \\ \\ 1 & \textit{for } i = N - M - 1, ..., 1 \end{cases}. \tag{39}$$

Proof. See the appendix section 8.D. □

As in the simple two creditor example presented in Section 2, expected delay under a collective action mechanism is independent of α and increasing in ϕ. This differs from uncoordinated bargaining where expected delay increases with α and is independent of ϕ. Generally, a collective action can increase or decrease delay depending on the relative sizes of the bargaining power and transaction costs parameters. Thus we are able to rationalize the ambiguous empirical findings of EICHENGREEN AND MODY (2000, 2004) and BECKER, RICHARDS AND THAICHAROEN (2003) of collective action clauses on borrowing costs.

5.D Secondary Debt Markets

Following the issuance of Brady bonds as part of the resolution of the 1980s debt crisis, a liquid market for developing country debt has developed. How does the ability to trade debt in secondary markets affect the incentive for market participants to consolidate or fragment debt holdings? Here we show that a well-functioning secondary debt market may either increase or decrease fragmentation depending on the level of creditor bargaining power.

With i creditors and before negotiations begin, each creditors claim is worth $\delta_{i-1}v_i$. Therefore, $\pi_i = i\delta_{i-1}v_i$ denotes the total value of debt with i players. Assuming that a strictly positive gain is required for a consolidation to take place, debt will be consolidated from i to $i-1$ creditors, or perhaps less that $i-1$ creditors, whenever $\pi_{i-1} < \pi_i$. In terms of our general notation, we have

$$\begin{aligned}
\pi_i - \pi_{i-1} &= i\delta_{i-1}v_i - (i-1)\,\delta_{i-2}v_{i-1} \\
&= i\beta_{i-1}\delta_{i-2}v_i - (i-1)\,\delta_{i-2}v_{i-1} \\
&\propto i\beta_{i-1}v_i - (i-1)\,v_{i-1}.
\end{aligned}$$

Substitution of

$$\beta_{i-1} = \frac{(i-1)v_{i-1}}{\sum_{j=1}^{i-1} v_j}$$

yields

$$\begin{aligned}
\pi_i - \pi_{i-1} &\propto i\frac{(i-1)v_{i-1}}{\sum_{j=1}^{i-1} v_j}v_i - (i-1)\,v_{i-1} \\
&\propto \frac{iv_i}{\sum_{j=1}^{i-1} v_j} - 1 \\
&\propto iv_i - \sum_{j=1}^{i-1} v_j,
\end{aligned}$$

or

$$\pi_i - \pi_{i-1} \propto iv_i - (i-1)\,\mu_{i-1}. \tag{40}$$

This expression tells us that there will be debt consolidation from i to $i-1$ creditors (or less) whenever $iv_i - (i-1)\,\mu_{i-1} < 0$. It also tells us that there will be debt dissolution from $i-1$ to i (or more) creditors whenever $iv_i - (i-1)\,\mu_{i-1} > 0$.[20] Generally speaking, the optimal number of creditors for any given initial number can be found as the solution, if it exists[21],

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[20] This does not contradict our standing assumption that $v_j < \mu_j$ since $\frac{i-1}{i} < 1$.

[21] A sufficient condition for existence of a solution is that there exists some $k \in \mathbb{N}$ such that $jv_j -$

to

$$\max_{i\in\mathbb{N}} \pi_i.$$

More can be said about debt consolidation if we adopt a constant share bargaining protocol. The following results show that the level of bargaining power is a key determinant of the optimal number of creditors in the market, with higher bargaining power being associated with greater consolidation.

Proposition 6. *There exists a monotonic strictly decreasing sequence* $\{\alpha_k\}_{k=1}^{\infty}$*, where* $\alpha_k \in (0,1)$*, such that for any initial number of creditors* i*, the optimal number of creditors is* j *whenever* $\alpha \in [\alpha_j, \alpha_{j-1}]$*.*

Proof. Consider $\pi_k - \pi_{k-1}$. Substitution of the terms for v_k and μ_{k-1} in (40) yields

$$\pi_k - \pi_{k-1} \propto k(1-\alpha)^{k-1} - \sum_{j=1}^{k-1} (1-\alpha)^{j-1} \text{ for } \alpha \in (0,1].$$

For convenience, define $\gamma = 1 - a$ for $a \in \mathbb{R}$, note that

$$\pi_k - \pi_{k-1} \propto f_k(\gamma) \equiv k\gamma^{k-1} - \gamma^{k-2} - \gamma^{k-3} - \cdots - \gamma - 1, \tag{41}$$

and allow $\gamma \in \mathbb{R}_+$. Note that for $a = \alpha \in (0,1]$, $\gamma \in [0,1)$. We have consolidation from k to (at most) $k-1$ creditors whenever $f_k(\gamma) < 0$, and dissolution from $k-1$ to (at least) k creditors whenever $f_k(\gamma) > 0$. Now consider the solutions to $f_k(\gamma) = 0$. Observe that since $f_k(0) = -1$, $f_k(1) = +1$ and f_k is continuous, there must exist some $\gamma' \in (0,1)$ such that $f_k(\gamma') = 0$. However, for the polynomial f_k there is one sign change in the sequence of coefficients of the powers of γ, as the exponent declines. Thus, Descartes' rule of signs tells us that there exists a single positive root, which we will denote as $\gamma_k > 0$. It follows from this and the previous observation (and that f_i is a function) that $\gamma_k = \gamma' \in (0,1)$, that $f_k(\gamma) < 0$ for $\gamma \in [0, \gamma_k)$, and that $f_k(\gamma) > 0$ for $\gamma \in (\gamma_k, 1]$.

$(j-1)\,\mu_{j-1} < 0$ for all $j \geq k$.

Now consider a rise in the number of creditors to $k+1$. We have

$$\begin{aligned} f_{k+1}(\gamma) &= (k+1)\gamma^{k} - \gamma^{k-1} - \gamma^{k-2} - \cdots - \gamma - 1 \\ &= \gamma^{k} + \gamma\left[k\gamma^{k-1} - \gamma^{k-2} - \gamma^{k-3} - \cdots - \gamma - 1\right] - 1 \\ &= \gamma^{k} + \gamma f_{k}(\gamma) - 1. \end{aligned}$$

Therefore,

$$f_{k+1}\left(\gamma_{k+1}\right) = \gamma_{k+1}^{k} + \gamma_{k+1} f_{k}\left(\gamma_{k+1}\right) - 1 = 0,$$

i.e.

$$f_{k}\left(\gamma_{k+1}\right) = \frac{1-\gamma_{k+1}^{k}}{\gamma_{k+1}} > 0,$$

from which it follows that $\gamma_{k+1} > \gamma_k$ for all k. This establishes a monotonic strictly increasing sequence $\{\gamma_k\}$.

Starting with an arbitrary number of creditors j, suppose that $\gamma \in \left(\gamma_k, \gamma_{k+1}\right)$. From above, it follows that $f_k(\gamma) > 0$ and $\pi_k > \pi_j$ for all $j < k$, and that $f_{k+1}(\gamma) < 0$ and $\pi_k < \pi_j$ for all $j > k$. Thus, k is the optimal number of creditors. Finally, note that since $\{\gamma_k\}$ is a strictly increasing sequence, $\{\alpha_k\}$ is a strictly decreasing sequence. □

First, suppose that we start with the optimal number of creditors j for a given level of creditor bargaining power in $[\alpha_j, \alpha_{j-1}]$. Then, the proposition demonstrates that as creditor bargaining power increases, the secondary market will act to (weakly) consolidate debt. The intuition for this is guided by the following observations. Note from the proof that

$$\pi_i - \pi_{i-1} \propto i\beta_{i-1}v_i - (i-1)\,v_{i-1}.$$

We showed in section 5.A that for fixed i and an increase in α, the strategic holdup effect is worsened, and β_{i-1} falls. It is straightforward to show that v_i falls for $\alpha > 1/i$.[22] Thus, as α rises, eventually $i\beta_{i-1}v_i$ must be lower, and it becomes profitable to consolidate debt.

Turning to the effect of the rise in α on *delay* (rather than on the effect on debt-holding), note that for a fixed number of creditors, strategic holdup tells us that delay tends

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[22] $\log(v_i) = \log\alpha + (i-1)\log(1-\alpha) + \log(V(1-\phi))$. Thus, $\log'(v_i) = \frac{1}{\alpha} - \frac{i-1}{1-\alpha} \propto \frac{1}{i} - \alpha$.

to increase. However, there is a countervailing effect, because higher α leads to consolidation of debt (and fewer debt-holders): In section 5.B, we showed that with fewer creditors, delay tends to fall. The effect of a rise in α on delay is determined by the net impact of these changes.

The proposition can be used to give us insights into the effect of the emergence of a secondary market for debt as follows. In practice, we cannot be sure that the number of creditors who initially purchase bonds coincides with the number allocated by the secondary market that emerges ex-post. If we start in a world where lending is dominated by few creditors, then debt will become more dispersed and delay will increase. Such a situation may describe the emergence of secondary markets during the era when debt was dominated by a few large banks. On the other hand, if the environment is initially one with a large number of lenders, then debt will be consolidated, and an emergent secondary market will cause delay to decrease.

So far, we have only characterized the optimal number of creditors, in terms of maximizing creditor value, without discussing the sort of mechanism through which consolidation or fragmentation might occur. Intuitively, one might think that secondary markets would operate to ensure that the market value of creditors claims were maximized. However, as noted in GROSSMAN AND HART (1980), a simple spot market for debt may not succeed in maximizing creditor value. The reason is a form of pecuniary externality. To see this, consider a simple example with three creditors. In the market with three creditors, a buyer who consolidates two units of debt exerts a positive externality on the remaining creditor, whose payoff rises from $\beta_2 v_3$ to v_2. Since each creditor who sells would like to receive this gain, a spot price must satisfy $p \geq \beta_2 v_3$. However, the total gain to the purchaser, v_2, is less than the cost of two units, $2p = 2\beta_2 v_3$, for some values of α (precisely, for α in the range $((5-\sqrt{13})/6, 1)$).[23]

A competitive market for the following contingent contract to buy debt will allow consolidation to be achieved. Consider the three creditor example. A buyer will offer to purchase each creditor's debt at price $p = \pi_2/3$ conditional on all creditors selling. If all

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[23] Note that $v_2 - 2\beta_2 v_3 \propto \alpha(1-\alpha)[3-2\alpha] < 0$ for $\alpha \in (0, \frac{2}{3})$, and $\pi_2 - \pi_3 > 0$ only for $\alpha \in (\frac{5-\sqrt{13}}{6}, 1)$.

creditors accept, then the buyer breaks even by purchasing all the debt and selling half of the total at price $\pi_2/2 = v_2$. Under such an arrangement, the (competitive) buyer breaks even since its return is $v_2 - 3p + v_2$. To see this, note that the first term v_2 is the return from retaining half the debt in a two creditor market. The second term $3p$ is the cost of purchase in the three creditor market. The final term v_2 is the (competitive) sale price of debt, which the buyer receives by selling half of its holding. If any single creditor refuses to sell then no gain is realized for any creditor and its payoff is $v_3 < \pi_2/3$. Thus all creditors will be willing to sell.

6 Vulture Creditors

In example 5 in section 3, we introduced a vulture creditor—who has either greater bargaining power or a lower discount rate and (possibly) higher bargaining costs. Our simple example supported the conventional idea that the presence of vultures would increase delay. Here, we generalize the example by allowing for many vultures and many normal creditors. We first consider equally patient creditors with different payoffs. Finally, we consider vultures with greater patience than normal creditors.

6.A Vultures as Legal Entrepreneurs

At any point, the game can be described by the numbers $m = 1, ..., M$ of normal and $n = 1, ..., N$ of vulture creditors—i.e. the pair (n, m) represents the 'state of the system'. Let $w_{m,n}$ be the normal and $v_{m,n}$ the vulture creditors' payoffs. For simplicity we assume strictly monotonic payoffs, i.e. they are decreasing in both m and n.

We will look for equilibria in which creditors of a given kind play symmetric mixed strategies. Consider the problem of a vulture creditor. Suppose all normal creditors mix according to CDF F and all the other vultures mix according to G. If the vulture enters at T its payoff is

$$\begin{aligned} EU &= v_{m,n} e^{-rT} \left[1 - F(T)\right]^m \left[1 - G(T)\right]^{n-1} \\ &\quad + v_{m-1,n} \int_0^T m f(t) \left[1 - F(t)\right]^{m-1} \left[1 - G(t)\right]^{n-1} e^{-rt} dt \\ &\quad + v_{m,n-1} \int_0^T (n-1) g(t) \left[1 - F(t)\right]^m \left[1 - G(t)\right]^{n-2} e^{-rt} dt. \end{aligned}$$

In a mixed strategy equilibrium, the vulture is indifferent between entering at T and $T+dT$, which yields

$$v_{m,n}\left(r+\frac{mf(T)}{1-F(T)}+\frac{(n-1)\,g(T)}{1-G(T)}\right)=v_{m-1,n}\frac{mf(T)}{1-F(T)}+v_{m,n-1}\frac{(n-1)\,g(T)}{1-G(T)}.$$

Similarly, for a normal creditor we get

$$w_{m,n}\left(r+\frac{(m-1)\,f(T)}{1-F(T)}+\frac{ng(T)}{1-G(T)}\right)=w_{m-1,n}\frac{(m-1)\,f(T)}{1-F(T)}+w_{m,n-1}\frac{ng(T)}{1-G(T)}.$$

Calculations analogous to the symmetric creditor case yield mixed strategies

$$f_{m,n}(t)=\lambda_{m,n}e^{-\lambda_{m,n}t}$$

for normal and

$$g_{m.n}(t)=\phi_{m,n}e^{-\phi_{m,n}t}$$

for vulture creditors, where

$$\phi_{m,n}=r\frac{(m-1)\,v_{m,n}\left(w_{m,n}-w_{m-1,n}\right)-mw_{m,n}\left(v_{m,n}-v_{m-1,n}\right)}{nm\left(v_{m,n}-v_{m-1,n}\right)\left(w_{m,n}-w_{m,n-1}\right)-(n-1)(m-1)\left(v_{m,n}-v_{m,n-1}\right)\left(w_{m,n}-w_{m-1,n}\right)},$$

and

$$\lambda_{m,n}=r\frac{(n-1)\,w_{m,n}\left(v_{m,n}-v_{m,n-1}\right)-nv_{m,n}\left(w_{m,n}-w_{m,n-1}\right)}{nm\left(v_{m,n}-v_{m-1,n}\right)\left(w_{m,n}-w_{m,n-1}\right)-(n-1)(m-1)\left(v_{m,n}-v_{m,n-1}\right)\left(w_{m,n}-w_{m-1,n}\right)}.$$

To calculate expected delay, we use the distribution of the order statistic for first engagement given m normal creditors playing according to F and n vultures playing according to G, i.e.

$$h_{m,n}(t)=ng(t)\left[1-G(t)\right]^{n-1}\left[1-F(t)\right]^{m}+mf(t)\left[1-F(t)\right]^{m-1}\left[1-G(t)\right]^{n}.$$

Substituting and rearranging gives

$$h_{m,n}(t)=\left(n\phi_{m,n}+m\lambda_{m,n}\right)e^{-\left(n\phi_{m,n}+m\lambda_{m,n}\right)t}.$$

The mean amount of time until first engagement is

$$Et_{m,n} = \frac{1}{n\phi_{m,n} + m\lambda_{m,n}}.$$

Similarly, the cost of delay is

$$\begin{aligned}\beta_{m,n} &= \int_0^\infty e^{-rt} h_{m,n}(t)\, dt \\ &= \frac{n\phi_{m,n} + m\lambda_{m,n}}{r + n\phi_{m,n} + m\lambda_{m,n}}.\end{aligned}$$

Recall that in the symmetric model with only normal creditors, we were able to interpret the interim discount factors as the cost of delay that exhausts all surplus from delay in expectation. Once we account for the asymmetric probabilities of engagement, the same is true in this model. To see why, note that the probability a specific normal creditor goes first, given n vultures and $m-1$ (other) normal creditors, is

$$\int_{s=0}^\infty \left[\int_{t=0}^s f_{m,n}(t)\, dt\right] h_{m-1,n}(s)\, ds,$$

where $f_{m,n}(t)$ is the normal creditor's density, and

$$h_{m-1,n}(s) = (n\phi + (m-1)\lambda)\, e^{-(n\phi + (m-1)\lambda)s},$$

is the density of the minimum over the choices of n vultures and $(m-1)$ normal creditors. Evaluating the integral yields

$$1 + \int_{s=0}^\infty (n\phi + (m-1)\lambda)\, e^{-(\lambda + n\phi + (m-1)\lambda)s} ds = \frac{1/\phi}{n/\lambda + m/\phi}.$$

To interpret, note that by the properties of the exponential pdf, $1/\phi$ is the mean time to concession of a vulture and $1/\lambda$ is the mean time to concession of a normal creditor. The longer a vulture creditor takes to concede, on average, the more likely a normal creditor concedes first. In the case that this normal creditor engages first, it receives $w_{m,n}$. Similarly, the probability that one of the other normal creditors goes first out of the remaining $m-1$

normals and n vultures is given by

$$\frac{(m-1)\lambda}{n\phi+(m-1)\lambda},$$

in which case the normal creditor under consideration earns $w_{m-1,n}$, while the probability that a vulture goes first is

$$\frac{n\phi}{n\phi+(m-1)\lambda},$$

in which case our normal creditor earns $w_{m,n-1}$.

Using the logic from the symmetric case, the cost of delay satisfies

$$\begin{aligned}\beta_{m,n} &= \frac{w_{m,n}}{\frac{\lambda}{n\phi+m\lambda}w_{m,n}+\frac{(m-1)\lambda}{n\phi+m\lambda}w_{m-1,n}+\frac{n\phi}{n\phi+m\lambda}w_{m,n-1}} \\ &= \frac{w_{m,n}(n\phi+m\lambda)}{\lambda w_{m,n}+(m-1)\lambda w_{m-1,n}+n\phi w_{m,n-1}}.\end{aligned}$$

The proof that these expressions are equivalent is given in the following proposition.

Proposition 7. *Consider two sequences of non-negative payoffs* $\{w_{m,n}\}$ *and* $\{v_{m,n}\}$ *for* $m = 1,...,M$ *and* $n = 1,.,,,,N$. *If both sequences are strictly monotonic, so that* $v_{m,n} < v_{m-1,n}$, $v_{m,n} < v_{m,n-1}$, $w_{m,n} < w_{m-1,n}$, *and* $w_{m,n} < w_{m,n-1}$,*for all* m *and* n, *then there is delay in making all concessions except the last, with expected discount factor*

$$\beta_{m,n} = \frac{w_{m,n}}{\frac{\lambda}{n\phi+m\lambda}w_{m,n}+\frac{(m-1)\lambda}{n\phi+m\lambda}w_{m-1,n}+\frac{n\phi}{n\phi+m\lambda}w_{m,n-1}}.$$

Proof. See the appendix section 8.E. □

We illustrate these results using the constant share bargaining protocol with different costs of bargaining.

Example 5. *Simple bargaining protocol with different bargaining power and costs*

With this protocol, payoffs are given by

$$w_{m,n} = \alpha^N\left(1-\alpha^N\right)^{m-1}\left(1-\alpha^V\right)^n V\left(1-\phi^N\right)$$

and

$$v_{m,n} = \alpha^V \left(1 - \alpha^N\right)^m \left(1 - \alpha^V\right)^{n-1} V \left(1 - \phi^V\right),$$

so that

$$\phi_{m,n} = r \frac{1 - \alpha^V}{\alpha^V (n + m - 1)}$$

and

$$\lambda_{m,n,} = r \frac{1 - \alpha^N}{\alpha^N (n + m - 1)}.$$

Substituting we find that the cost of delay is given by

$$\beta_{m,n} = \frac{n\alpha^N + m\alpha^V - (n + m)\, \alpha^N \alpha^V}{n\alpha^N + m\alpha^V - \alpha^N \alpha^V}.$$

It is easy to see that $\beta_{m,n}$ is decreasing in both α^N and α^V. Hence, if the number of vulture creditors increases, keeping the total number of creditors constant, delay increases. Moreover, as delay is independent of the bargaining costs of both parties, the result holds even if vulture creditors face higher bargaining costs.

6.B Vultures as Patient Negotiators

Another dimension along which vultures may differ from other creditors is their discount rate—vultures are often regarded as more patient investors. To capture this case, suppose that $v_{m,n} = w_{m,n} = w_{m+n}$ for all m and n, so that $w_{m,n-1} = w_{m-1,n}$, and let $r^V < r^N$ denote the discount rates of both types of creditors. The above process yields equilibrium mixed strategy profiles of the form $f_{m,n}(t) = \lambda_{m,n} e^{-r\lambda_{m,n} t}$ for the normal creditors and $g_{m.n}(t) = \phi_{m,n} e^{-r\phi_{m,n} t}$ for the vulture creditors, where

$$\lambda_{m,n} = -\frac{w_{m,n}}{w_{m,n} - w_{m-1,n}} \left[\frac{r^V n - r^N (n-1)}{nm - (n-1)(m-1)}\right]$$

and

$$\phi_{m,n} = -\frac{w_{m,n}}{w_{m,n} - w_{m-1,n}} \left[\frac{r^N m - r^V (m-1)}{nm - (n-1)(m-1)}\right].$$

Using our formulae above, the expected delay is given by

$$Et_{m,n} = \frac{(n+m-1)}{n+m} \frac{1}{\frac{n}{n+m} r^V + \frac{m}{n+m} r^N} \frac{w_{m-1,n-} w_{m,n}}{w_{m,n}}.$$

Compared to the case where all agents have the discount rate—and delay is given by

$$Et_{m+n} = \frac{(n+m-1)}{n+m} \frac{w_{m-1,n-} w_{m,n}}{r w_{m,n}},$$

we now evaluate delay using a *weighted average* of discount rates. Moreover, if vultures are more patient than normal creditors, so that $r^V < r^N$, expected delay increases for any given payoff schedule.

Example 6. *Simple bargaining protocol with different degrees of patience*

Recall that for the simple bargaining protocol, we had

$$v_i = \alpha (1-\alpha)^{i-1} V (1-\phi),$$

so that in this case we obtain

$$Et_{m,n} = \frac{(n+m-1)}{n+m} \frac{1}{\frac{n}{n+m} r^V + \frac{m}{n+m} r^N} \frac{\alpha}{1-\alpha}.$$

7 Conclusion

Sovereign debt negotiation is time consuming, taking on average more than seven years to complete. A number of recent high profile restructurings suggest that these delays are due to collective action problems among creditors. With significant increases in the number and diversity of creditors in recent years, the rise of liquid secondary bond markets, and the proliferation of 'vulture' creditors, it is likely that collective action problems will become more prevalent in the future. Consequently, there is a great deal of interest among policymakers in policies to encourage creditors—or even force them—to improve coordination.

Our paper develops a model of negotiation in a weak contractual environment that reflects the key features of the environment in which sovereign debt is negotiation. The model generates delay in equilibrium as a result of what we term a 'strategic holdup' effect, where each creditor prefers to bargain with the sovereign after others have settled in order to exploit bilateral bargaining power. The paper conducts a positive economic analysis of the effect on delay in different sovereign debt environments. We derive reasonable conditions under which an increase in the number of creditors exacerbates the strategic holdup and increases delay. Similarly, we show that the advent of vulture creditors is likely to increase delay. Collective action policies, on the other hand, have an ambiguous effect because, even while limiting the ability of creditors to engage in strategic holdout, they exacerbate free-rider problems. Finally, we show that secondary bond markets induce greater consolidation the more bargaining power creditors possess, and greater dispersion of ownership as bargaining power falls. Thus, the secondary market increase delay with 'weak' creditors, and decreases delay if creditors are 'strong'.

It is important to stress that our paper considers the case of a sovereign that is already in default. It abstracts from the sovereign's decision to borrow and perhaps default in the first instance. Our analysis is also positive in nature, and so does not address some important welfare questions. Finally, our analysis does not take a stand on the quantitative magnitudes of the effects we identify. In a related paper (PITCHFORD AND WRIGHT 2007) we use numerical methods to study the quantitative effect of collective action problems in a related environment. We find that the model of that paper is able to match a number of features of the observed distribution of delays in restructuring. We also model the ex-ante borrowing-and-default stage of the sovereign's decision, adding this to the beginning of the numerical model of restructuring. This allows us to study the effect of the introduction of collective action policies on the borrowing and default decisions of countries, as well as on their welfare.

8 Appendix

8.A Lemma 8 and proof

The first step is to view the game as one with i players and only two payoffs—the amount u_i received by entering immediately, if no others also try to enter—and the amount EU_{i-1}. The latter is the continuation payoff from subgame $i-1$, but for the purposes of the first step in our solution can be seen as a fixed payoff. The following lemma summarizes the the solution to the two-payoff game:

Lemma 8. *Consider the game with i players, current value payoffs u_i for the first creditor to engage, EU_{i-1} for the other $i-1$ players and a symmetric tie-breaking rule.*

1. *In the one-player game the optimal strategy is immediate entry with $EU_1 = v_1$, and $\beta_1 = 1$.*
2. *If $u_i \geq EU_{i-1}$ then $F_i^*(0) = 1$,*

$$EU_i = \frac{(i-1)EU_{i-1} + u_i}{i}, \tag{42}$$

   and $\beta_i = 1$.

3. *Suppose $u_i < EU_{i-1}$.*
   - *(a) There exists a symmetric mixed-strategy equilibrium*

$$F_i^*(t) = 1 - \exp\left\{-\frac{ru_i}{(i-1)[EU_{i-1} - u_i]}t\right\} \tag{43}$$

     with expected payoff

$$EU_i = u_i, \tag{44}$$

     and expected discount factor

$$\beta_i = \frac{iu_i}{(i-1)EU_{i-1} + u_i}. \tag{45}$$

   - *(b) There exist pure strategy equilibria where one player enters immediately with probability 1 and $i-1$ other players enter immediately after the first. The first entrant receives u_i, the others receive EU_{i-1}, and $\beta_i = 1$.*

Proof. The proof of parts 1 and 2 are trivial and are presented in section 4.B. Part 3 with $u_i < EU_{i-1}$ is proven as follows. Recall the indifference condition (13) in section 4.B:

$$u_i = u_i e^{-rT}[1 - F_i(T)]^{i-1} + EU_{i-1}\int_0^T e^{-r\tau_{\mathrm{m}}} g_{i-1}(\tau_{\mathrm{m}})d\tau_{\mathrm{m}}, \tag{46}$$

where τ_{m} is the minimum engagement time of $i-1$ creditors who each play according to F_i.

The density of the order-statistic τ_{m} is

$$g_{i-1}(t) = (i-1)f_i(t)[1-F_i(t)]^{i-2},$$

which on substitution into (46) yields

$$u_i = u_i e^{-rT}[1-F_i(T)]^{i-1} + EU_{i-1}\int_0^T e^{-rt}(i-1)f_i(t)[1-F_i(t)]^{i-2}dt. \tag{47}$$

Since the player is indifferent between engaging at any instant, the derivative of the RHS of (47) with respect to T will be zero, i.e.

$$-ru_i e^{-rT}[1-F_i(T)]^{i-1} - [u_i - EU_{i-1}]e^{-rT}f_i(T)(i-1)[1-F_i(T)]^{i-2} = 0. \tag{48}$$

Re-arranging and cancelling terms yields the differential equation

$$\frac{f_i}{1-F_i} = \frac{1}{i-1}\frac{ru_i}{EU_{i-1}-u_i}.$$

Note that the LHS is the derivative of $-\log(1-F_i)$, which means

$$-\frac{d}{dt}\log(1-F_i) = \frac{1}{i-1}\frac{ru_i}{EU_{i-1}-u_i},$$

which upon integrating from 0 to t yields

$$\int_0^t d\log(1-F_i) = -\int_0^t \frac{1}{i-1}\frac{ru_i}{EU_{i-1}-u_i}dt.$$

With initial condition $F_i(0)=0$ we have

$$\log(1-F_i) = -\frac{1}{i-1}\frac{ru_i}{EU_{i-1}-u_i}t.$$

This yields (43), i.e.

$$F_i^*(t) = 1 - e^{-\frac{q_i}{i-1}t}, \tag{49}$$

and

$$f_i^*(t) = \frac{q_i}{i-1}e^{-\frac{q_i}{i-1}t}, \tag{50}$$

where

$$q_i \equiv \frac{ru_i}{EU_{i-1}-u_i}. \tag{51}$$

Since the creditor is indifferent between playing F_i^* and choosing any pure strategy, the payoff with the symmetric mixed strategy equilibrium is $EU_i = u_i$. Thus, all of the potential gains $EU_{i-1} - u_i$ are dissipated due to delay caused by competition between the players.

The expected discount factor β_i in (45) is the expected value of e^{-rt}, conditional on entry by the first to engage in subgame i. Thus, it is calculated using the density for the minimum entry time of i random variables drawn from the equilibrium density F_i^*, which we denote

$$g_i(t) \equiv i f_i^*(t)[1 - F_i^*(t)]^{i-1} = \frac{iq_i}{(i-1)} \exp\left\{-\frac{iq_i}{i-1}t\right\} \tag{52}$$

as follows:

$$\begin{aligned} \beta_i &= \int_0^\infty e^{-rt} g_i(t)dt \\ &= \frac{i}{(i-1)} q_i \int_0^\infty \exp\left\{-\frac{iq_i + (i-1)r}{i-1}t\right\} dt \\ &= \left[-\frac{iq_i}{iq_i + (i-1)r} \exp\left\{-\frac{iq_i + (i-1)r}{i-1}t\right\}\right]_0^\infty \\ &= \frac{iu_i}{(i-1)EU_{i-1} + u_i}. \end{aligned}$$

This completes the proof of part 3(a).

Finally, for part 3(b), a pure strategy equilibrium also exists, which is necessarily asymmetric. Suppose the $i-1$ other creditors adopt the strategy "Enter immediately after the creditor has entered, otherwise do not enter". The creditor's strategy is to enter immediately. Clearly, she will not deviate, since doing so results in an indefinite delay. No other creditor will deviate because by tie-breaking it will receive the lower payoff u_i with positive probability. □

8.B Proof of Proposition 1

The general case with many prizes follows from a careful interpretation of the payoffs in Lemma 8. Payoff EU_i is what is received by the first creditor to engage with the sovereign, whether it be by rationing when $u_i \geq EU_{i-1}$, or by being the first conceder when $u_i < EU_{i-1}$. Using parts 2 and 3(a) of Lemma 8, it is immediate that

$$EU_i = \min\left\{u_i, \frac{(i-1)EU_{i-1} + u_i}{i}\right\}. \tag{53}$$

To calculate β_i, first consider the case $EU_i = u_i < EU_{i-1}$. Note that equation (45) in part 3(a) of Lemma 8 yields

$$\beta_i = \frac{iEU_i}{(i-1)EU_{i-1} + iu_i}.$$

Now consider the case $u_i \geq EU_{i-1}$. By (53),

$$EU_i = \frac{(i-1)EU_{i-1} + u_i}{i}.$$

Therefore, if we define

$$\beta_i = \frac{iEU_i}{(i-1)EU_{i-1} + u_i}$$

substitution yields $\beta_i = 1$ as required. The formula (15) therefore nests both cases. The CDF is found by direction substitution, similarly to finding β_i, by applying cases 2 and 3(a) of Lemma 8. The solution for subgame $i+1$ is identical, and since we have the solution for $i = 1$, the complete solution is characterized by the proposition.

8.C Proof of Proposition 2

The proof is by induction. Suppose the expression (19) for β_i and (21) for EU_i are correct for $i = j-1$, so that

$$\beta_{j-1} = \frac{v_{j-1}}{\mu_{j-1}}$$

and $EU_{j-1} = \delta_{j-2} v_{j-1}$. Using equation (14) from proposition 1, and substituting $u_j = \delta_{j-1} v_j$, we have

$$\begin{aligned}
EU_j &= \min\left\{\delta_{j-1} v_j, \frac{(j-1)\,\delta_{j-2} v_{j-1} + \delta_{j-1} v_j}{j}\right\} \\
&= \delta_{j-2} \min\left\{\beta_{j-1} v_j, \frac{(j-1)\, v_{j-1} + \beta_{j-1} v_j}{j}\right\} \\
&= \delta_{j-2} \min\left\{\frac{v_{j-1}}{\mu_{j-1}} v_j, \frac{(j-1)\, v_{j-1} + \frac{v_{j-1}}{\mu_{j-1}} v_j}{j}\right\} \\
&= \frac{\delta_{j-2} v_{j-1}}{\mu_{j-1}} \min\left\{v_j, \frac{(j-1)\,\mu_{j-1} + v_j}{j}\right\} \\
&= \delta_{j-1} \min\left\{v_j, \mu_j\right\}.
\end{aligned}$$

By assumption, $v_j < \mu_j$ so that $EU_j = \delta_{j-1}v_j$ as required. Substitution into equation (15) of proposition 1 yields

$$\begin{aligned} \beta_j &= \frac{j\delta_{j-1}v_j}{(i-1)\delta_{j-2}v_{j-1} + \delta_{j-1}v_j} \\ &= \frac{jv_j}{(i-1)\frac{v_{j-1}}{\beta_{j-1}} + v_j} \\ &= \frac{jv_j}{(i-1)\mu_{j-1} + v_j} \\ &= \frac{v_j}{\mu_j}, \end{aligned}$$

as required. To complete this part of the proof, note that (19) and (21) hold for $i = 2$. From (14) we have

$$EU_2 = \min\left\{v_2, \frac{v_1 + v_2}{2}\right\} = v_2,$$

and from (15) we have

$$\beta_2 = \frac{2v_2}{v_1 + v_2} = \frac{v_2}{\mu_2}.$$

Finally, note that from the proof of lemma 8 above, the density for the minimum settlement time out of i random variables is given by equation (52) and (51). Therefore, the expected time until the first settlement in subgame i is

$$\begin{aligned} Et_i &= \frac{i-1}{iq_i} \\ &= \frac{i-1}{i}\frac{EU_{i-1} - u_i}{ru_i} \\ &= \frac{i-1}{i}\frac{\delta_{i-2}v_{i-1} - \delta_{i-1}v_i}{r\delta_{i-1}v_i} \\ &= \frac{i-1}{i}\frac{v_{i-1} - \beta_{i-1}v_i}{r\beta_{i-1}v_i} \\ &= \frac{i-1}{i}\frac{v_{i-1} - \frac{v_{i-1}}{\mu_{i-1}}v_i}{r\frac{v_{i-1}}{\mu_{i-1}}v_i} \\ &= \frac{i-1}{i}\frac{\mu_{i-1} - v_i}{rv_i}, \end{aligned}$$

as required.

8.D Proof of Proposition 5

To calculate the expected discount factor for each subgame, note that once the agreement is made, the remaining creditors receive α^*V with no delay (i.e. $\beta_i^{cN} = 1$, $i =$

$N-M-1,...,1$). Thus, the sequence of payoffs does not satisfy $v_i^{cN} < \mu_i^{cN}$ (with obvious notation), so we cannot use proposition 2 directly to calculate β_i^{cN} for $i = N,...,N-M$. However we can use lemma 8 and proposition 1: Note that

$$v_{N-M-1} = v_{N-M-2} = ... = v_1 = \alpha^* V$$

and

$$v_N = v_{N-1} = ... = v_{N-M} = \alpha^* V \left(1-\phi\right)$$

so that equation (45) yields

$$\begin{aligned} \beta_{N-M} &= \frac{i v_{N-M}}{(i-1)v_{N-M-1} + v_{N-M}} \\ & \frac{\left(N-M\right)\alpha^* V \left(1-\phi\right)}{\left(N-M-1\right)\alpha^* V + \alpha^* V \left(1-\phi\right)} \\ &= \frac{\left(N-M\right)\left(1-\phi\right)}{\left(N-M-1\right) + \left(1-\phi\right)}. \end{aligned}$$

Applying the recursive procedure given by proposition 1, we have, for $j = 1,...,M$ that

$$\begin{aligned} \beta_{N-M+j} &= \frac{\left(N-M+j\right)\delta_{N-M+j-1}\alpha^* V \left(1-\phi\right)}{\left(N-M+j-1\right)\delta_{N-M+j-2}\alpha^* V \left(1-\phi\right) + \delta_{N-M+j-1}\alpha^* V \left(1-\phi\right)} \\ &= \frac{\left(N-M+j\right)\beta_{N-M+j-1}\delta_{N-M+j-2}}{\left(N-M+j-1\right)\delta_{N-M+j-2} + \beta_{N-M+j-1}\delta_{N-M+j-2}} \\ &= \frac{\left(N-M+j\right)\beta_{N-M+j-1}}{\left(N-M+j-1\right) + \beta_{N-M+j-1}}. \end{aligned}$$

Thus for $j = 1$,

$$\begin{aligned} \beta_{N-M+1} &= \frac{\left(N-M+1\right)\beta_{N-M}}{\left(N-M\right) + \beta_{N-M}} \\ &= \frac{\left(N-M+1\right)\frac{(N-M)(1-\phi)}{(N-M-1)+(1-\phi)}}{\left(N-M\right) + \frac{(N-M)(1-\phi)}{(N-M-1)+(1-\phi)}} \\ &= \frac{\left(N-M+1\right)\left(1-\phi\right)}{\left(N-M-1\right) + 2\left(1-\phi\right)}, \end{aligned}$$

and by recursion for arbitrary j,

$$\beta_{N-M+j} = \frac{\left(N-M+1\right)\left(1-\phi\right)}{\left(N-M-1\right) + \left(j+1\right)\left(1-\phi\right)}.$$

Rewriting the index as $i = N - M + j$ we have the result

$$\begin{aligned}\beta_i &= \frac{(N-M+1)(1-\phi)}{(N-M-1)+(i-(N-M-1))(1-\phi)} \\ &= \frac{(N-M+1)(1-\phi)}{(N-M-1)\phi + i(1-\phi)}.\end{aligned}$$

8.E Proof of Proposition 7

As shown in the text,

$$\beta_{m,n} = \frac{n\phi_{m,n} + m\lambda_{m,n}}{r + n\phi_{m,n} + m\lambda_{m,n}}.$$

We aim to show that

$$\begin{aligned}\beta_{m,n} &= \frac{w_{m,n}}{\frac{\lambda_{m,n}}{n\phi_{m,n}+m\lambda_{m,n}}w_{m,n} + \frac{(m-1)\lambda_{m,n}}{n\phi_{m,n}+m\lambda_{m,n}}w_{m-1,n} + \frac{n\phi_{m,n}}{n\phi_{m,n}+m\lambda_{m,n}}w_{m,n-1}} \\ &= \frac{w_{m,n}\left(n\phi_{m,n}+m\lambda_{m,n}\right)}{\lambda_{m,n}w_{m,n} + (m-1)\lambda_{m,n}w_{m-1,n} + n\phi_{m,n}w_{m,n-1}}.\end{aligned}$$

The proof will be complete if we can show that

$$(m-1)\lambda_{m,n}\frac{w_{m-1,n}}{w_{m,n}} + n\phi_{m,n}\frac{w_{m,n-1}}{w_{m,n}} = r + n\phi_{m,n} + (m-1)\lambda_{m,n},$$

or that

$$(m-1)\lambda_{m,n}\left(\frac{w_{m-1,n}-w_{m,n}}{w_{m,n}}\right) + n\phi_{m,n}\left(\frac{w_{m,n-1}-w_{m,n}}{w_{m,n}}\right) = r.$$

But if we let

$$\Phi = nm\left(v_{m,n}-v_{m-1,n}\right)\left(w_{m,n}-w_{m,n-1}\right) - (n-1)(m-1)\left(v_{m,n}-v_{m,n-1}\right)\left(w_{m,n}-w_{m-1,n}\right),$$

then using the definitions of $\lambda_{m,n}$ and $\phi_{m,n}$

$$\begin{aligned}&(m-1)\lambda_{m,n}\left(\frac{w_{m-1,n}-w_{m,n}}{w_{m,n}}\right) + n\phi_{m,n}\left(\frac{w_{m,n-1}-w_{m,n}}{w_{m,n}}\right) \\ =\ &\frac{r}{\Phi}\left(nm\left(v_{m,n}-v_{m-1,n}\right)\left(w_{m,n}-w_{m,n-1}\right) - (n-1)(m-1)\left(v_{m,n}-v_{m,n-1}\right)\left(w_{m,n}-w_{m-1,n}\right)\right) \\ =\ &r,\end{aligned}$$

as required.

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